Properties of spherical triangle:

1. If a circle is drawn on a sphere so that the radius of the circle is the same as the radius of the sphere it is called a great circle. Any other circle is a small circle.

2. An infinite number of great circles can pass through one point, but only one great circle can pass through two points, unless they are diametrically opposite.

3. A spherical triangle is a triangle each of whose sides is a great circle.

4. The length of the arc of a circle can be measured by the angle which the arc subtends at the center of the circle. The sides of a spherical triangle are measured in degree, minutes, and seconds.

5. No sides of a spherical triangle can therefore exceed 180°.

6. The three angles of a spherical triangle must together be more than 180° and less than 540°.

7. The greater side is opposite the greater angle, if two sides are equal their opposite angles are equal.

8. If one angle of the triangle is 90° it is called a right-angle triangle, and if one side of the triangle 90° it is called a quadrant triangle.

Solving right angle spherical triangle:

In case giving a spherical triangle in which one angle is 90°, we use the following fundamental rules,

\[
\sin \text{middle} = \tan(\text{adj.}) \times \tan(\text{adj.}) \\
\sin \text{middle} = \cos(\text{opp.}) \times \cos(\text{opp.})
\]
Example:

1. Solve the following right angle spherical triangle ABC given that: \( a = 87^\circ 16' \), \( B = 38^\circ 45' \), \( C = 90^\circ \)

Solution:

In the first

\[
\sin a = \tan b \cdot \tan (90 - B)
\]
\[
\therefore \sin a = \tan b \cdot \cot B
\]
\[
\therefore \tan b = \frac{\sin a}{\cot B} = \sin a \cdot \tan B = 0.801671751
\]
\[
\therefore b = \tan^{-1} 0.801671751 = 38.718657 = 38^\circ 43' 5''
\]

In the second

\[
\sin (90 - A) = \cos a \cdot \cos (90 - B)
\]
\[
\therefore \cos A = \cos a \cdot \sin B
\]
\[
\therefore \cos A = 0.029848771
\]
\[
\therefore A = \cos^{-1} 0.02984877145 = 88^\circ 17' 22.33''
\]

In the third

\[
\sin (90 - B) = \tan a \cdot \tan (90 - c)
\]
\[
\therefore \cos B = \tan a \cdot \cot c
\]
\[
\therefore \tan c = \frac{\tan a}{\cos B} = 26.85778049
\]
\[
\therefore c = \tan^{-1} 26.85778049 = 87^\circ 52' 3''
\]
Solving right angle spherical triangle:

1) Solve the following right angle spherical triangle ABC given that:
\[
\sin \text{middle} = \tan(\text{adj.}) \times \tan(\text{adj.})
\]
\[
\sin \text{middle} = \cos(\text{opp.}) \times \cos(\text{opp.})
\]

Lecture

1. \(A = 90^\circ\), \(c = 46^\circ 18' 30"\), \(B = 34^\circ 27' 30"\)
2. \(C = 90^\circ\), \(c = 69^\circ 25' 11"\), \(B = 63^\circ 25' 03"\)
3. \(a = 85^\circ 17'\), \(b = 102^\circ 26' 15"\), \(B = 90^\circ\)
4. \(C = 90^\circ\), \(a = 120^\circ 18' 45''\), \(b = 101^\circ 9'\)
5. \(A = 90^\circ\), \(B = 100^\circ\), \(C = 87^\circ 10'\)

Section

1. \(c = 61^\circ 4' 56"\), \(a = 40^\circ 31' 20"\), \(C = 90^\circ\)
2. \(b = 70^\circ 23' 42"\), \(B = 90^\circ\), \(c = 48^\circ 39' 16"\)
3. \(A = 90^\circ\), \(b = 100^\circ\), \(c = 98^\circ 20'\)
4. \(A = 90^\circ\), \(c = 100^\circ 42'\), \(B = 78^\circ 10'\)

Home work

1. \(C = 90^\circ\), \(A = 66^\circ 07' 20"\), \(a = 59^\circ 28' 27"\)
2. \(A = 90^\circ\), \(B = 72^\circ 19'\), \(b = 50^\circ 50'\)
3. \(B = 90^\circ\), \(a = 25^\circ 12' 48"\), \(c = 52^\circ 0' 45"\)
4. \(A = 90^\circ\), \(c = 46^\circ 12'\), \(a = 70^\circ 49'\)
5. \(c = 78^\circ 53' 20"\), \(A = 83^\circ 56' 40"\), \(C = 90^\circ\)