ARAB ACADEMY FOR SCIENCE, TECHNOLOGY & MARITIME TRANSPORT

Probability and Statistics Sheet

BA203 - BA325 - BA326 - BA327 - BA329

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COLLEGE OF ENGINEERING & TECHNOLOGY

BASIC & APPLIED SCIENCE DEPARTMENT

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Index

CHAPTER 1: STATISTICS

Some useful rules

$$Mean = \bar{x} = \frac{\sum x_i f_i}{N}, N = \sum f_i$$

$$Median = L_1 + \left(\frac{\frac{N+1}{2} - \sum_{m-1} f_i}{f_m}\right) w$$

$$Mode = L_1 + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) w$$

$$Variance = S^2 = \frac{1}{N-1}\left[\sum x_i^2 f_i - N\bar{x}^2\right], \qquad S.D = S = \sqrt{variance}$$

$$C.V. = 100 * (S/\bar{x}), \qquad Skewness = \frac{\bar{x} - mode}{S}$$

Exercises

1. In the following data represents the weight of 40 male students in a certain college recorded to the nearest bound.

138	161	164	150	132	144	125	149	150	157
146	145	158	140	147	136	148	152	156	144
168	135	126	138	177	163	118	154	145	165
146	142	173	142	147	135	153	140	128	135

a. Construct the class intervals for the previous data.
b. A percentage frequency distribution.
c. A histogram and a frequency polygon.
d. A relative frequency distribution.
e. Ascending and descending cumulative frequency distribution and sketch the curve.
f. Find mean, median, mode, variance, standard deviation, coefficient of variation and measures of symmetry (skewness).

2. The following are the body weight in grams of 50 rats used in a study of vitamin deficiencies:

136	106	92	115	118	121	137	132	120	95
109	125	114	119	115	101	129	87	108	148
110	117	133	135	126	127	103	110	126	113
113	102	82	104	137	120	95	146	126	124
119	146	114	105	132	126	118	100	113	129

a. Construct a frequency distribution table with the classes 80 - 89, 90 -99...
b. A percentage frequency distribution.
c. A histogram and a frequency polygon.
d. A relative frequency distribution.

e. Ascending and descending cumulative frequency distribution and sketch the curve.
f. Find mean, median, mode, variance, standard deviation, coefficient of variation and measures of symmetry (skewness).

3. The following table shows the scores on an intelligence test by a group of 80 children.

Scores	80-89	90-99	100-109	110-119	120-129	130-139
No. of children	5	8	12	20	25	10

a. Construct the cumulative frequency distributions and plot them, then deduce the value of the median from the graph.
b. Calculate the median and check your result with that obtained in **(a)**.
c. Construct the histogram and the polygon shape.

4. The following table shows the ages of employee's working in the ministry of education:

Age	20 – 24	25 - 29	30 - 34	35 - 39	40 - 44	45 - 49	50 - 54
No. of employees	15	10	12	13	20	14	16

a. Construct the descending cumulative frequency distribution and plot it, then deduce the value of the median from the graph.
b. What is the percentage of employees whose age's arc between 42 and 52?
c. Calculate the median and check your results with that obtained in **(a)**.
d. Calculate the skewness and comment on the result obtained.

5. The mean of marks obtained by 20 students in class A is 75 and the mean of marks obtained by 30 students in class B is 70, find the mean of all marks taken by the 50 students.

6. How are mean and median affected when it is known that for a group of 10 students scoring an average of 60 marks and median of 62 marks, the best paper was wrongly marked 90 instead of 85.

7. Consider the following observations 8, 9, 12, 13, 8 and 4 find the mean, median, mode and standard deviation.

8. The ages of people in a class (to nearest year) are as follows:

Age	18	19	20	21	22	23	24
No. of students	5	10	14	16	10	3	2

a. Find the range, median and mode.
b. Calculate the mean and the coefficient of variation.

9. The mean speeds of a sample of ships were recorded as they passed through a traffic separation scheme. The results are as shown in the frequency table below:

Speed (knots)	No. of ships	Relative frequency
5–9	10	0.25
10-14	...	...
15-19	...	0.30
20-24	6	...
25-29	...	0.05

a. Complete the bland cells in the given table.
b. Calculate the mean, the mode and the coefficient of variation.

10. The following frequency table gives hemoglobin levels (in g/dl) of patients:

Hemoglobin levels (g/dl)	Frequency	Relative frequency
8 - 10.9	8	0.20
11 - 13.9	**...**	**...**
14 - 16.9	**...**	0.30
17 - 19.9	6	**...**
20 - 22.9	**...**	0.10

1. Complete the blank cells in the given table.
2. Calculate the mean, the median and the coefficient of variation.

Against each statement, put a tick mark (√) if it is TRUE and a (X) if it is FALSE:

1. In constructing frequency distribution tables the number of classes is direct proportional with the range. ( )

2. The standard deviation is measured in the same units as the observation in the data set.( )

3. The mean is affected by extreme values. ( )

4. The mode is always found. ( )

5. If it is known that for a group of 16 students scoring an average of 70 marks, the best paper was wrongly marked 90 instead of 74, then the correct mean is 69. ( )

Circle the correct answer from each of the following multiple choice questions:

1. The standard deviation of the observations 11, 13, 17, 18 and 21 equals:

i. 2 **ii.** 4 **iii.** 16 **iv.** 64

2. If the mean of x_1, x_2, x_3 and x_4 is 10 , then the mean of $5x_1 + 2$, $5x_2 + 2$, $5x_3 + 2$ and $5x_4 + 2$ is

i. 10 **ii.** 15 **iii.** 50 **iv.** 52

3. Given the following frequency distribution:

x_i	**-5**	**0**	**5**	**10**	**15**
f_i	4	7	14	15	10

The mean

i. 0.5 **ii.** 7 **iii.** 7.5 **iv.** None of above

4. For the frequency distribution given the in question 3. The median is

i. 5 **ii.** 7.6 **iii.** 10 **iv.** None of above

5. The value of every observation in the data set is taken into account when calculate:

i. Mean **ii.** Median **iii.** Mode **iv.** Range

CHAPTER 2: PROBABILITY

Some useful rules

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(\bar{A}) = 1 - P(A)$$
$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$
$$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$$
$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Total probability: $P(A) = \sum_{i=1}^{n} P(A/B_i)\, P(B_i)$

Bayes' formula: $P(A/B) = \frac{P(B/A)P(A)}{P(B)}$

Exercises

- ***Probability Concepts***

1. Let A, B and C be there arbitrary events. Find expressions for the following events that of A, B and C.

a. Only A occurs,
b. All three events occur,
c. Non occurs,
d. At least one occur,
e. Not more than two occur,
f. At least two occur,
g. One and only one occurs,

2. Set C consists of the citizens of a certain town who voted "YES" for water fluoridation. Set D of consists of the citizens of the same town who have preschool children. Define:

a. $C \cap \bar{D}$ **b.** $\bar{C} \cap D$ **c.** $\bar{C} \cup \bar{D}$

3. Of 50 patients on the third floor of a hospital, 35 are female and 12 are over 70 years of age. Among those over 70 years of age, eight are female. How many of the 50 patients are female or not over 70 years of age? (*Use a Venn diagram to help you answer the question*).

4. When an experiment is performed, one and only one of the events A_1, A_2 or A_3 wil occur. Find $P(A_1), P(A_2),$ and $P(A_3)$ under each of the following assumptions:

a. $P(A_1) = P(A_2) = P(A_3)$.
b. $P(A_1) = P(A_2)$ and $P(A_3) = \frac{1}{2}$.
c. $P(A_1) = 2P(A_2) = 3\, P(A_3)$.

5. **Explain why there must be a mistake in each of the following statements:**

a. The probability that Jean will pass the bar examination is 0.66 and the probability that she will not pass is 0.34.

b. The probability that the home team will win an upcoming football game is 0.77, the probability it will tie the game is 0.08, and the probability that it will win or tie the game is 0.95.

c. The probabilities that a secretary will make 0, 1,2,3,4 or 5 or more mistakes in typing a report are respectively 0.12, 0.25, 0.36, 0.14, 0.09 and 0.07.

d. The probabilities that a bank will get 0, 1, 2, 3 or more, bad checks on any given day are respectively 0.08, 0.21, 0.29 and 0.40.

6. Let $S = \{e_1 , e_2 , e_3 , e_4 \}$ be a sample space and let P be probability function defined on S.

a. Find$P(e_1)$ and $P(e_2)$ if $P(e_3) = P(e_4) = \frac{1}{4}$and $P(e_1) = 2P(e_2)$.

b. Find $P(e_1)$ if $p\{ e_2 , e_3\} = \frac{2}{3}$,$P\{ e_3 , e_4\} = \frac{1}{2}$and $P(e_2) = \frac{1}{3}$.

7. A probability experiment has sample space S={a,b,c,d,e} where P(a)=0.2, P(b)=0.15, P(c)=0.08, and the probability of d is twice the probability of e. Find P(d) and P(e).

8. The probabilities that the serviceability of a new X-ray machine will be rated very difficult, difficult ,average, easy, or very easy are respectively 0.12, 0.17, 0.34, 0.29, 0.08 Find the probabilities that the serviceability or the machine will be ruled
   a. Difficult or very difficult
   b. Neither very difficult nor very easy.

9. A police department needs new tires for its patrol cars and the probabilities are 0.15, 0.24, 0.03, 0.28, 0.22 and 0.08 that it will buy Uniroyal tires, Goodyear tires, Michelin tires, General tires, Goodrich tires, or Armstrong tires. Find the probabilities that it will buy
   a. Goodyear or Goodrich tires
   b. Uniroyal, Michelin, or Goodrich tires
   c. Michelin or Armstrong tires
   d. Uniroyal, Michelin, General or Goodrich tires

10. A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P (G), where G is the event that a number greater than 3 occurs on a single roll of the die.

11. There are 15 balls, numbered from 1 to 15 in a bag. If a person selects one at random, what is the probability that the number printed on the ball will be

a. A prime number greater than 5.

b. An odd number less than 11.

12. In a high school graduation class of 100 students, 54 studied mathematics, 69 studied English and 35 studied both mathematics and English. If one of these students is selected at random, find the probability that:

a. The student took mathematics or English,
b. The student didn't take either of these subjects,
c. The student took English but not mathematics.

13. A coin is tossed twice. What is the probability that at least one head occurs?

14. A coin is weighted so that the probability that a head appears is twice as likely as tail. Find the probability that we get head.

15. An integer between 1 and 100 is selected at random, find the probability of getting a perfect square if

a. All integers are equally likely to be selected,
b. All integers between 1 and 50 are twice as likely to occur as the rest.

16. If A and B are two events, $P(\overline{A}) = 0.35, P(\overline{A} \cap \overline{B}) = 0.2$ and $P(A \cap \overline{B}) = 0.5$ Then Calculate the Probability of :

a. Occurrence of both A and B.
b. Occurrence of only A.
c. Nonoccurrence of B.
d. Occurrence of only A or only B.

17. If A and B are mutually exclusive, P(A) = 0.37 and P(B) = 0.44, find

a. $P(\bar{A})$
b. $P(A \cup B)$
c. $P(\bar{A} \cap B)$
d. $P(\overline{B})$
e. $P(A \cap \overline{B})$
f. $P(\overline{A} \cap \overline{B})$

18. Three students A, B and C are in a swimming race. A and B have the same probability of winning and each is twice as likely to win as C.

a. What is the probability that A does not win?
b. What is the probability that B or C wins?

19. In a certain population of women 4% have had breast cancer, 20% are smokers and 3% are smokers and have had breast cancer. A woman is selected at random from the population. What is the probability that:

a. She has had breast cancer or smokes?
b. She has had breast cancer and she is not a smoker?

- ***<u>Conditional probability</u>***

20. A coin is biased so that a head is twice as likely to occur as tail. If the coin is tossed twice, what is the probability of getting:

a. Exactly 2 tails
b. A tail and a head?

21. Show that the axioms of probability are satisfied by conditional probabilities. In other words, show that if P(B) > 0, then

a. $P(A/B) \geq 0$; **b.** $P(B/B) = 1$;

c. $P(A_1 \cup A_2 \cup \ldots/B) = P(A_1/B) + \ldots$ for any sequence of mutually exclusive events $A_1, A_2, \ldots$

22. There are 90 applicants for a job with the news department of a television station some of them are college graduates and some are not, some of them have at least three years' experience and some have not, with the .exact breakdown being.

	College Graduate	Not college graduates
At least three years experience	18	9
Less than three years experience	36	27

If the order in which the applicants are interviewed by the station manager is random, G is the event that the first applicant interviewed is a college graduates, and T is the event that the first applicant interviewed as at least three years' experience determine each of the following probabilities directly from the entries and the row and column totals of the table:

a. $P(G)$ **b.** $P(\overline{T})$ **c.** $P(G \cap T)$

d. $P(\overline{G} \cap T)$ **e.** $P(T/G)$ **f.** $P(\overline{G}/T)$.

23. Given three events A, B and C such that $P(A \cap B \cap C) \neq 0$ and $P(C/A \cap B) = P(C/B)$, show that $P(A/B \cap C) = P(A/B)$.

24. Show that if $P(B/A)=P(B)$ and $P(B) \neq 0$, then $P(A/B) = P(A)$.

25. Let A and B events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{7}{12}$ Find $P(A/B)$, $P(A/\bar{B})$, $P(\bar{A}/B)$ and $P(\bar{A}/\bar{B})$.

26. Find P(B/A) in the following cases

a. If A and B are mutually exclusive. **b.** If A is a subset of B.

c. If B is a subset of A.

27. The probability that a student, selected at random from certain College, will pass a certain economics exam is 0.8 and will pass in both economics and religion exams are 0.5. What is the probability that he will pass religion if it is known that he had passed economics?

28. A box of fuses contains 20 fuses, of which 5 are defective. If 3 of the fuses are selected at random and removed from the box in succession without replacement, what is the probability that all three fuses are defective?

29. In a certain college 25% of the students failed mathematics, 15% of the students failed chemistry and 10% of the students failed both mathematics and chemistry. A student is selected at random:

a. If the student passes Math., what is the probability that he also passes Chemistry?
b. If the student failed chemistry, what is the probability that he passes Math.?
c. What is the probability that he passed one and only one course?

30. The probability that a student, selected at random from a certain college, passes mathematics is 0.8, the probability that passes English is 0.6 and the probability of passing at least one of them is 0.9.

a. If the student passed Math., what is the probability that he also passes English?
b. If the student failed Math., what is the probability that he also failed English?
c. What is the probability that he failed both Math., and English?

31. The probability that a regularly scheduled flight departs on time is 0.83, the probability that it arrives on time is 0.82, and the probability that it departs and arrives on time is 0.78. Find the probability that a plane,

a. Arrives on time given that it departed on time,
b. Depart on time given that it has arrived on time.

32. The probability of surviving a certain transplant operation is 0.55. If a patient survives the operation, the probability that his or her body will reject the transplant within a month is 0.20. What is the probability of surviving both of these critical stages?

33. A number is selected at random from {1, 2, . . ., 100}. Given that the number selected is divisible by 2, find the probability that it is divisible by 3.

34. A number is selected at random from {1, 2, . . ., 100}. Given that the number selected is divisible by 2, find the probability that it is divisible by 3 or 5.

35. There are n persons in a room.

a. What is the probability that at least two persons have the same birthday?
b. Calculate this probability for n = 50.
c. How large need n be for this probability to be greater than 0.5?

- ***Independent events***

36. If the events A,B, and C are independent, show that

a. A and B ∩ C are independent. **b.** A and BUC are independent.

37. Let A and B events with P(A) = $\frac{1}{4}$, P(AUB) =$\frac{1}{3}$ and P(B) = p

a. Find p if A and B are mutually exclusive.
b. Find p if A and B are independent.
c. Find p if A is a subset of B.

38. Let **A** and **B** be two independent events in S. It is known that **P(A ∩ B)** = 0.16 and **P(A ∪ B)** = 0.64. Find P(A) and P(B).

39. Let A and B two events with $P(A) = \frac{1}{4}$, $P(\, A \cup B) = \frac{1}{3}$ and $P(B) = p$.

a. Find p if A and B are mutually exclusive.
b. Find p if A and B are independent.

40. If A, B, and C are events such that P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{4}$, and P(C) = $\frac{1}{5}$. find P(AUBUC) under each of the following assumptions:

a. If A, B, and C are mutually exclusive.
b. If A, B, and C are independent.

41. A coin is tossed three times and the eight possible outcomes, HHH, HHT, HTH, HTT, THH, THT, TTH and TTT, are assumed to be equally likely. If A is the event that a head occurs on each of the first two tosses, B is the event that a tail occurs on the third toss, and C is the event that exactly two tails occur in the three tosses, show that

a. Events A and B are independent.
b. Events B and C are dependent.

42. From a box containing 5 black balls and 3 green balls, 2 balls are drawn in succession, the first ball being replaced in the box before the second draw is made.

a. What is the probability that both balls are the same color?
b. What is the probability that each color is represented?

43. From a box containing 5 black balls and 3 green balls, 2 balls are drawn in succession **without replacement**; i.e. the second draw is made without replacing of the first ball in the box.

a. What is the probability that both balls are the same color?
b. What is the probability that each color is represented?

44. A box contains 8 red, 3 white and 9 blue balls. If 3 balls are drawn at random **without replacement**, determine the probability that

a. all 3 are red,
b. all 3 are white,
c. One of each color is drawn.
d. 2 are red and 1 is blue,
e. at least 1 is white,

45. The probability that three men hit a target is respectively $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{3}{8}$ each shoots once at the target.

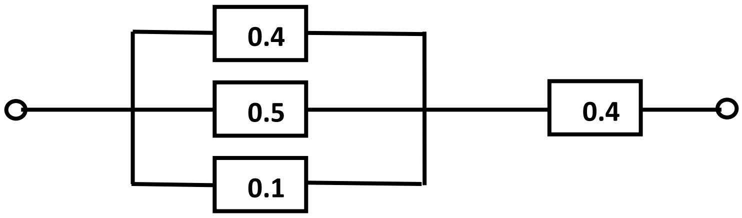
a. Find the probability that exactly one of them hits the target.
b. If only one hit the target, what is the probability that it was the first man?

46. In an experiment consisting of *10* throws of a pair of fair dice, find the probability of the event that at least one double 6 occurs.

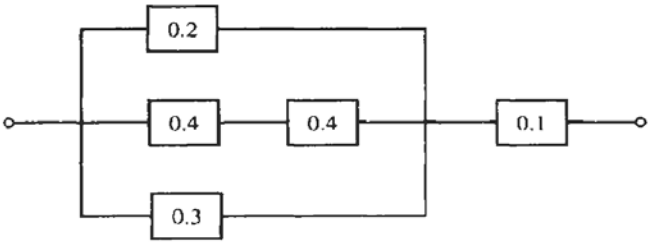
47. A and B play 12 games of chess of which 6 are won by A, 4 are won by B, and 2 end in a tie. They agree to play a tournament consisting of 3 games. Find the probability that

a. A wins all the three games,
b. Two games end in a tie.
c. A and B wins alternately,
d. B wins at least one game.

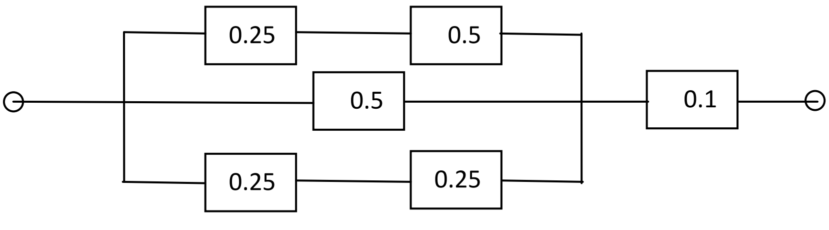
48. The relay network shown in the following figure operates if and only if there is a closed path of relays from left to right. Assume that relays fail independently and that the probability of failure of each relay is as shown. What is the probability that the relay network operates?



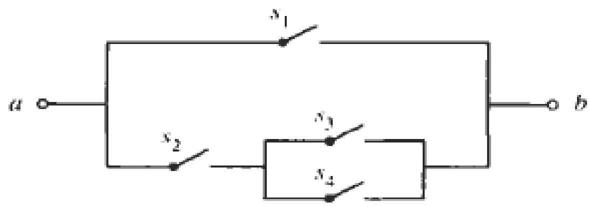
49. The relay network shown in the figure operates if and only if there is a closed path of relays from left to right. Assume that relays fail independently and that the probability of failure of each relay is as shown. What is the probability that the relay network **will not operate**?



50. The relay network shown in the following figure operates if and only if there is a closed path of relays from left to right. Assume that relays fail independently and that the probability of failure of each relay is as shown. What is the probability that the relay network operates?



51. Consider the switching network shown in the following figure. It is equally likely that a switch will or will not work. Find the probability that a closed path will exist between terminals a and b.



- ***Total probability and Bayes' formula***

52. The members of a consulting firm rent cars from three rental agencies: 60% from agency 1, 30 % from agency 2, and 10 % from agency 3. If 9% of the cars from agency 1 need a tune-up, 20 % of the cars from agency 2 need a tune-up, and 6% of the cars from agency 3 need a tune-up, what is the probability that a rental car delivered to the firm will need a tune-up?

53. On a fighter plane 3 single-round shots are fired. The probability of success for the first shot is 0.4, the second 0.5 and the third 0.7. It is sufficient to fire 3 shots in order to destroy the plane. There is a probability of 0.2 for one successful shot and 0.6 for two successful shots to destroy the plane. After these 3 shots the plane is destroyed what is the probability that it is destroyed by only one successful shot?

54. At an electronic plant, it is known from past experience that the probability is 0.84 that a new worker who has attended the company's training program will meet the production quota, and that the corresponding probability is 0.49 for a new worker who has not attended the company's training program. If 40% of all new workers attend the training program, what is the probability that a new worker will meet the production quota?

55. A factory has three machines X, Y and Z produce plastic gears. The output of machine X is 1.5 times the output of machine Y and the same as the output of machine Z. The percentages of defective output of these machines are respectively 2%, 5% and 3%. A gear is selected at random and is found to be defective, find the probability that it was produced by machine Y.

56. We are given three similar boxes (equally likely) of microchips as follows:
Box B1 contains 22 microchips, of which 7 are defective,
Box B2 contains 33 microchips, of which 15 are defective ,
Box B3 contains 44 microchips, of which 12 are defective .
An experiment consists of choosing a box at random then selecting a microchip from the box. If the component obtained is defective, find the probability that it came from Box 3.

57. Three kinds of machines A, Band C produce .respectively 30%, 30% and 40% of the total number of bulbs in a factory. The percentages of defective outputs of these machines are 5%, 4% and 3%. If a bulb is selected at random, find the probability that this bulb is defective. If the bulb is defective what is the probability that it is produced by machine C.

58. An aircraft emergency locator transmitter (ELT) is a device designed to transmit a signal in the case of a crash. The Altigauge Manufacturing Company makes 80% of the ELTs, the Bryant Company makes 15% of them, and the Chartair Company makes the other 5%. The ELTs made by Altigauge have a 4% rate of defects, the Bryant ELTs have a 6% rate of defects, and the Chartair ELTs have a 9% rate of defects. If an ELT is randomly selected and tested, find the probability that it was manufactured by the Bryant Company if the test indicates that the ELT is defective.

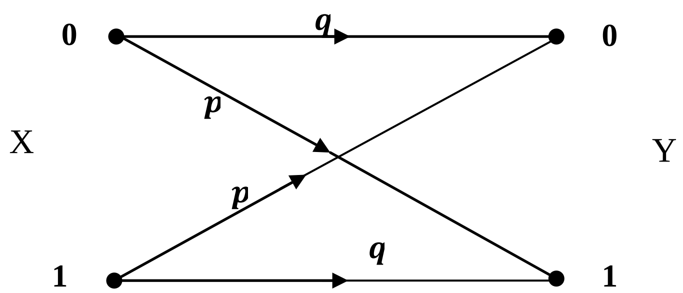
59. Suppose that a laboratory test to detect a certain disease has the following statistics.
Let A = event that the tested person has the disease
B = event that the test result is positive
It is known that P(B/A) = 0.99 and P(B/A') = 0.005
and 0.1 percent of the population actually has the disease. What is the probability that a person has the disease given that the test result is positive?

60. Consider the binary communication channel shown in Figure. The channel input symbol X may assume the state 0 or the state 1, and, similarly, the channel output symbol Y may assume either the state 0 or the state 1. Because of the channel noise, an input 0 may convert to an output 1 and vice versa. The channel is characterized by the channel transition probability p_0 , q_0 , p_1 and q_1 , defined by

$$p_0 = P(y_1 / x_0) \quad \text{and} \quad p_1 = P(y_0 / x_1)$$

$$q_0 = P(y_0 / x_0) \quad \text{and} \quad q_1 = P(y_1 / x_1)$$

where x_0 and x_1 denote the events (X = 0) and (X = 1) respectively , and y_0 and y_1 denote the events (Y = 0) and (Y = 1) respectively. Note that $p_0 + q_0 = 1$ and $p_1 + q_1 = 1$. Assuming that $P(x_0) = 0.5$, $p_0 = 0.1$ and $p_1 = 0.2$.



a. Find $P(y_0)$ and $P(y_1)$.
b. If a 0 was observed at the output , what is the probability that a 0 was the input state.
c. If a 1 was observed at the output , what is the probability that a 1 was the input state.
d. Calculate the probability of error P_e .

- ***<u>Techniques of counting (Enumeration methods)</u>***

61. The number of different arrangements or permutations consisting of 3 letters each which can be formed from the 7 letters A,B,C,D,E,F,G ?

62. The number of different permutations of the 11 letters of the word MISSISSIPPI, which consists of 1 M, 4 I's, 4 S’s and 2 P's ?

63. How many 4 digit numbers can be formed with the 10 digits 0, 1, …….., 9 if:

a. Repetitions are allowed. **b.** Repetitions are not allowed.

c. The last digit must be zero and repetitions are not allowed.

d. It is an even number and repetitions are not allows.

64. It is required to seat 4 boys and 4 girls in a row. How many arrangements are possible if:

a. The boys and girls seat alternately. **b.** Only the girls have to seat together.

65. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

66. Four different mathematics books, six different physics books, and two different chemistry books are to be arranged on a shelf. How many different arrangements are possible if

a. the book in each particular subject must all stand together,
b. Only the math books must stand together?

67. A student is to answer 7 out of 10 questions on an examination. How many choices have he, if he must answer at least 3 of the first 5 questions?

68. Five red marbles, 2 white marbles, and 3 blue marbles are arranged in a row. If all the marbles of the same color are not distinguishable from each other, how many different arrangements are possible?

69. In how many ways can a group of 10 people be divided into

a. Two groups consisting of 7 and 3 people,
b. Three groups consisting of 5, 3 and 2 people.
c. Two groups of 5 people.

70. Out of 5 mathematicians and 7 physicists, a committee consisting of 2 mathematicians and 3 physicists is to be formed. In how many ways this be done if

a. Any mathematician and any physicist can be included.
b. One particular physicist must be in the committee.
c. Two particular mathematicians cannot be on the committee

71. The number of ways in which 3 cards can be chosen or selected from a total of 8 different cards?

72. Two cards are randomly drawn from a deck of 52 playing cards. Find the probability that both cards will be greater than 3 and less than 8.

73. In a poker game five cards are dealt at random from an ordinary deck of 52 playing cards. Find the probabilities of getting.

a. Two pairs (any two distinct face values occurring exactly twice)
b. Four of a kind (four cards of equal face value)

74. Crates of eggs are inspected for blood clots by randomly removing three eggs in succession and examining their contents. If all three eggs are good the crate is shipped otherwise it will be rejected. What is the probability that a crate will be shipped if it contains 120 eggs of which 10 have blood clots?

75. In a football team of size 30, 5 are freshmen, 10 are sophomores, 12 are juniors and 3 are seniors. If 8 players are selected at random, what is the probability that the sample includes exactly 2 students from each class?

76. Box 1 contains the letters B, C, D, E, F. Box 2 contains the letters W, X, Y, and Z. How many five letters code words are possible using 3 letters from box 1 and 2 letters from box 2?

77. Say that there are 3 defective items in o lot of 50 items. A sample of size 10 is taken at random and without replacement. Find the probability that the sample contains

a. exactly one defective item
b. at most 2 defective items.

<u>Circle the correct answer from each of the following multiple choice questions:</u>

1- Find p if A is a subset of B. If $P(\mathrm{A/B}) = \frac{1}{2}$, $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = 0.4$ and $\mathrm{P}\left(\overline{\mathrm{A}}\right) = 0.8$ then P(B) is equal to :

a. 0.2 **b.** 0.5 **c.** 0.4 **d.** 0.1

2- If $\mathrm{P}(A) = 0.7$, $\mathrm{P}(B) = 0.4$ and $\mathrm{P}(\mathrm{A}/\mathrm{B}) = 0.28$, then $\mathrm{P}(B/\overline{A})$ is :

a. 0.16 **b.** 0.48 **c.** 0.49 **d.** 0.96

3- If $\mathrm{P(A)} = \mathrm{P}(\overline{B}) = 0.6$ and $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = 0.76$, then A and B are:

a. $A \subset B$ **b.** Independent **c.** dependent **d.** mutually exclusive

4- If $P(\bar{A}/B) = 1$, then A & B are:
a. independent **b.** $B \subset A$ **c.** $A \subset B$ **d.** mutually exclusive

5- If $P(A/\bar{B}) = 0$, then A & B are:
a. independent **b.** $B \subset A$ **c.** $A \subset B$ **d.** mutually exclusive

6- If P(A) = P(B) = 0.5 , and $P(A \cup B) = 0.75$, then A and B are :
a. independent **b.** dependent **c.** $A \subset B$ **d.** mutually exclusive

7- If P(A/B) =1 , then A and B are :
a. independent **b.** $B \subset A$ **c.** $A \subset B$ **d.** mutually exclusive

8- If $P(\bar{A}) = 0.6$ and $P(\bar{A} \cap B) = 0.5$, then the probability of occurrence of neither A nor B is :
a. 1/2 **b.** 0.05 **c.** 0.1 **d.** none of the above

Against each statement, put a tick (√) if it is TRUE and a (×) if it is FALSE:

1- If P(A) = 0.4 , P(B) = 0.5 and A & B are mutually exclusive events then

$P(A \cap B) = 0.9$. ( )

2- If P (A) = 0.8, P (B) = 0.4 and $P(A \cap \bar{B}) = 0.5$, then $P(A \cup B) = 0.9$. ( )

3- If $P(\bar{A}) = 0.6$ and $P(\bar{A} \cap B) = 0.5$ then the probability of neither A nor B is equal to 0.1. ( )

4- If P(A/B) =1 , then A and B are independent events . ( )

5- The two event A and B are said to be independent events if and only if $P(A \cup B) = P(A) + P(B)$. ( )

6- $P(A / \bar{B}) = 0.5$, if P(A) = 0.65 , P(B) = 0.3 and $P(A \cap B) = 0.3$. ( )

CHAPTER 3: RANDOM VARIABLES

Some useful rules

Discrete Random Variable	***Continuous Random Variable***
P.m.f. $f(x) = P(X = x)$ where $f(x) \geq 0$ and $\sum_{\forall x_i} f(x_i) = 1$. **CDF** $F(x) = P(X \leq x) = \sum_{i: x_i \leq x} f(x_i)$ CDF must satisfy the following: $F(-\infty) = 0$; $F(\infty) = 1$, if $a < b$ then $F(a) \leq F(b)$ $F(x)$ is continuous from the right. $E(X^r) = \sum x^r f(x)$	**P.m.f.** $P(a < x < b) = \int_a^b f(x)\, dx$ where $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x) dx = 1$. **CDF** $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt$ CDF must satisfy the following: $F(-\infty) = 0$; $F(\infty) = 1$, if $a < b$ then $F(a) \leq F(b)$ $F(x)$ is continuous from the right. $E(X^r) = \int_{-\infty}^{\infty} x^r f(x)\, dx$

Note:
$P(a < x \leq b) = F(b) - F(a)$
$E(a X + b) = a E(X) + b = a\, \mu_X + b$
$Var\,(X) = E(X^2) - [E(X)]^2 = \sigma_X^2$
$Var(aX + b) = a^2 Var(X) = a^2 \sigma^2$

Exercises

Discrete Random Variable

1. For each of the following, determine whether the given function can serve as the probability mass function of a r.v. with the given range.
   a. $f(x) = \frac{x-2}{5}$, for $x = 1, 2, 3, 4, 5$;
   b. $f(x) = \frac{x^2}{30}$, for $x = 0, 1, 2, 3, 4$.
   c. $f(x) = \frac{1}{5}$, for $x = 0, 1, 2, 3, 4, 5$.

2. For each of the following, determine the constant c so that the function can serve as the probability mass function of a r.v. with the given range.
   a. $f(x) = c$, for $x = 1, 2, 3, 4, 5$.
   b. $f(x) = c\binom{5}{x}$,for$x = 0, 1, 2, 3, 4, 5$.
   c. $f(x) = c\left(\frac{1}{4}\right)^{x}$,for $x = 1, 2, 3, \dots \dots ..$
   d. $f(x) = c^2$,for $x = 1, 2, 3, 4, 5 \dots \dots \dots , n$.

3. For each of the following, determine whether the given values can serve as the values of a distribution function of a random variable with the range x = 1, 2, 3, 4
   a- F(1) = 0.3,F(2) =0.5, F(3) =0.8 and F(4)= 1.2
   b- F(1) = 0.5 , F(2) = 0.4,F(3) = 0.7 and F (4) = 1.0
   c- F(1) = 0.25, F(2) = 0.61. F(3) = 0.83 and F(4) = 1.0

4. For what values of k can $f(x) = (1-k)k^x$serve as the p.m.f. of a r.v. with countable infinite range $x = 0, 1, 2, 3, 4, \dots$?

5. If the discrete r.v. has the CDF,

$$\mathrm{F}(x) = \begin{cases} 0 & \text{for } x < -1 \\ \frac{1}{4} & \text{for } -1 \le x < 1 \\ \frac{1}{2} & \text{for } 1 \le x < 3 \\ \frac{3}{4} & \text{for } 3 \le x < 5 \\ 1 & \text{for } x \ge 5 \end{cases}$$

   Find
   a. $\mathrm{P}(-0.4 < X < 3.6)$.
   b. $\mathrm{P}(\mathrm{X} = 5)$.
   c. The p.m.f of X.

6. If X has a distribution function

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{1}{3} & \text{for } 1 \le x < 4 \\ \frac{1}{2} & \text{for } 4 \le x < 6 \\ \frac{5}{6} & \text{for } 6 \le x < 1O \\ 1 & \text{for } x \ge 10 \end{cases}$$

   Find
   a. The probability distribution of r.v X
   b. $\mathrm{P}(\mathrm{X} = 4)$.
   c. $\mathrm{P}(2 < X < 6)$.
   d. Mean, variance and standard deviation.
   e. $\mathrm{Var}(1 + 4\mathrm{X})$.

7. If a discrete r.v X has the CDF

$$F(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{9} & -2 \le x < -1 \\ \frac{3}{9} & -1 \le x < 0 \\ \frac{6}{9} & 0 \le x < 1 \\ \frac{8}{9} & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

Find:

a. The p.m.f.

b. $E(-2X - 1)$ and $Var(3 - 2X)$.

c. $P(X < 0)$ and $P(-0.5 \le X \le 1.5)$.

8. If a discrete r.v X has the p.m.f

x	-3	-2	-1	0	1	2	3
$f(x)$	1/25	2/25	4/25	11/25	4/25	a	1/25

Find:

a. The value of a

b. the CDF of X,

c. $E(X + 4)$ and $var(-2X + 1)$

d. $P(-1.5 \le X \le 2.2)$.

9. If the probability distribution for the discrete r.v. X is given by :

x	-2	-1	0	**a**	2
$f(x)$	$\frac{1}{9}$	$\frac{2}{9}$	**b**	$\frac{2}{9}$	$\frac{1}{9}$

Find:

a. The value of a and b such that E[X] = 0.

b. The variance.

c. The CDF.

d. $P(X > -1)$ and $P(-1.5 < X < 2)$.

10. A shipment of 8 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If X is the number of defective sets purchased by the hotel, find the probability distribution of X. Express the results graphically as a probability histogram.

11. A coin is biased so that heads are twice as likely as tails. For three independent tosses of the coins, find the probability distribution of X, the total number of heads and represent it graphically. Find also the CDF of X and use it to find $P(1 < X \leq 3)$ and $P(X > 2)$. Find the probability distribution of Y, the number of heads minus the number of tails.

12. A man with n keys wants to open his door and tries the keys independently and at random. Find the expected value and variance of the number of trials if:

a. Unsuccessful keys are not eliminated from further selection;

b. They are elimination (Assuming that only one key fits the door).

13. A lot of 12 T.V. sets includes 2 with white cords. If 3 of the sets are chosen at random for shipment to a hotel, how many sets with white cords can the shipper expect to send to the hotel?

<u>Continuous Random Variable</u>

14. The p.d.f. of the r.v. X is given by:

$$f(x) = \begin{cases} \mathrm{x} & \text{for } 0 < x < 1 \\ 2 - \mathrm{x} & \text{for } 1 \leq \mathrm{x} < c \\ 0 & \text{otherwise} \end{cases}$$

Find:

a. The value of c.

b. The CDF of X.

c. P(0.8 < X < 1.2).

d. Mean and variance of X.

15. The p.d.f. of the r.v. X is given by,

$$f(x) = \begin{cases} \mathrm{kx} & \text{for } 0 < x < 1 \\ \mathrm{k} & \text{for } 1 < x \leq 2 \\ \mathrm{k}(3 - \mathrm{x}) & \text{for } 2 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the constant k, sketch the graphs of the p.d.f.

b. Find the CDF of X, sketch the graph

c. Find $\mathrm{P}(0.8 < X < 1.2)$ using the p.d.f. and the CDF.

16. Consider the following cumulative distribution function of the random variable X

$$F_X(x) = \begin{cases} 0 & x < -1 \\ 0.5(1+x^3) & -1 < x < 1 \\ 1 & x > 1 \end{cases}$$

Find:

a. The probability density function of X

b. $P(-1.5 < X \leq 1)$

c. E(X) and Var(X)

17. In a certain city the daily consumption of water (in millions of liters) is a r.v whose p.d.f. is given by

$$f(x) = \begin{cases} \frac{1}{9} x e^{-x/3} & \text{for } x > 0 \\ 0 & otherwise \end{cases}$$

What are the probabilities that on a given day

a. The water consumption in this city is no more than 6 million liters;

b. The water supply is inadequate if the daily capacity of this city is 9 million liters.

18. The CDF. of the r.v. X is given by,

$$F(x) = \begin{cases} 1 - (1+x)\mathrm{e}^{-x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

Find

a. $\mathrm{P}(\mathrm{X} < 2)$, $\mathrm{P}(1 < X < 3)$, $\mathrm{P}(\mathrm{X} > 4)$

b. the p.d.f. of X.

19. The p.d.f of the r.v. X is given by :

$$f(x) = \begin{cases} k\,x(1-x)^2 & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Find:

a. The value of k

b. The CDF

c. E (X) and Var (2 – 3 X)

d. P (0.3 < X < 1.2)

20. The p.d.f of the r.v. is given by :

$$f(x) = \begin{cases} kx^2(1-x) & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Find:

a. The value of k

b. The CDF

c. E (X) and Var(3 – 4X)

d. P (-1 < X < 0.7)

21. The p.d.f. of the r.v. X is given by:

$$f(x) = \begin{cases} \mathrm{kx}(1 - \mathrm{cx}^2) & 0 \le \mathrm{x} \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Show that $c < 1$ and that $k = \frac{4}{2-c}$.
b. For the case $c = 1$, Find $E(X)$, $\mathrm{Var}(2X - 3)$
c. $\mathrm{P}(0.2 < X < 0.8)$.

22. The p.d.f of the r.v X is given by

$$f(x) = \begin{cases} k\,x^2 & 0 < x \le 1 \\ k\,x & 1 < x < 2 \\ k\,(4 - x) & 2 \le x \le 3 \\ 0 & otherwise \end{cases}$$

Find:
a. The value of k
b. The CDF
c. E (X) , Var (1- 4X) and P (1.8 < X < 3.2).

23. The p.d.f of r.v X is given by

$$f(x) = \begin{cases} k(1 - x^2) & |x| < 1 \\ 0 & o.w. \end{cases}$$

Find
a. The value of K
b. C.D.F
c. E (X)
d. Var (1-5 X)
e. P (-1.5 < X < 0.5)

24. The p.d.f. of the r.v. X is given by

$$f(x) = \begin{cases} kx(1 - x) & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

Find :
a. the value of k
b. the CDF of X.
c. Find $\mathrm{E}(X)$
d. $Var(1 - 2X)$
e. $P(0.2 < X < 0.8)$.
f. Find the median M of X (Hint: M is defined by $P(X > M) = P(X < M) = 0.5$)

Moment Generating Functions

25. Explain why there can be no r.v. for which $M_X(t) = \frac{t}{1-t}$.

26. Find the M.G.F of the discrete r.v. X w hich has the p.m.f.

$$f(x) = 2\left(\frac{1}{3}\right)^x \quad for\ x = 1, 2, 3, \dots \dots$$

And use it to determine the mean and variance of X.

27. Given the moment generating function $M(t) = e^{5t+2t^2}$ of a r.v. X, find the M.G.F. of the r.v. $= \frac{X-5}{2}$, and use it to find mean variance of X.

28. The p.d.f. of the r.v. X is given by

$$f(x) = ke^{-|x|}, -\infty < x < \infty$$

- **a.** Find the constant k.
- **b.** Find the corresponding CDF, then evaluate $P(-1 < X < 2)$.
- **c.** Show that the moment generating function of X is given by $M(t) = \frac{1}{1-t^2}$, -1<t<1
- **d.** Calculate the mean and the standard deviation of X.

29. The p.d.f of r.v Z is given by

$$f(z) = ke^{-2|z|} \qquad -\infty < z < \infty$$

- **a.** The constant k.
- **b.** C. D. F then evaluate P(-2 < Z < 4)
- **c.** The moment generating function.
- **d.** Mean and standard deviation

30. Let $R_X = \ln M_X(t)$. Show that $R_X'(0) = \mu$ and $R_X''(0) = \sigma^2$.

31. Find the M.G.F. of r.v. X whose p.d.f. is given by

$$f(x) = \begin{cases} e^{-x} & for\ x > 0 \\ 0 & otherwise \end{cases}$$

And hence find mean and variance.

CHAPTER 4: SPECAIL PROBABILITY DISTRIBUTIONS

Table of distributions

Name	Param.	PMF or PDF $f(x) =$	Mean	Variance
Bernoulli	BER(p)	$P(X = 1) = p, P(X = 0) = q$	p	pq
Binomial	BIN(n, p)	$\binom{n}{x} p^x q^{n-x}$, $x = 0, 1, 2, \dots, n$	np	npq
Geometric	GEOM(p)	pq^{x-1} , $x = 1,2,3, \dots \dots$	$\frac{1}{p}$	$\frac{q}{p^2}$
NegBinom	NBIN(k,p)	$\binom{x-1}{k-1} p^k q^{x-k}$, $x = k, k+1, k+2, \dots$	$\frac{k}{p}$	$\frac{k\,q}{p^2}$
Hypergeom	HYP(n,N,M)	$\frac{\binom{M}{x}\binom{N}{n-x}}{\binom{M+N}{n}}$	$\frac{n\,M}{N}$	$\frac{n\,M(N-M)(N-n)}{N^2\,(N-1)}$
Poisson	POIS(λ)	$\frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, 3, \dots ..$	λ	λ
Uniform	U(α, β)	$\frac{1}{\beta - \alpha}$, $x \in (\alpha, \beta)$	$\frac{\beta + \alpha}{2}$	$\frac{(\beta - \alpha)^2}{12}$
Exponential	EXP(θ)	$\frac{1}{\theta}\, e^{-\frac{x}{\theta}}$, $x > 0$ **0** o.w	θ	θ^2
Normal	N(μ, σ^2)	$\frac{1}{\sigma\sqrt{2\pi}}\, e^{-(x-\mu)^2/(2\sigma^2)}$	μ	σ^2

Note that:

Geometric: $P(X > a) = q^a$

Uniform CDF: $F(x) = \begin{cases} 0 & , x \le \alpha \\ \frac{x-\alpha}{\beta-\alpha} & , \alpha < x < \beta \\ 1 & , x \ge \beta \end{cases}$

Exponential: $P(X > t + a \,/ X > t) = P(X > a)$

Exponential CDF: $F(x) = \begin{cases} 1 - e^{-\frac{x}{\theta}} & , x > 0 \\ 0 & , o.w \end{cases}$

Normal to standard normal: $Z = \frac{X-\mu}{\sigma}$

Exercises

Discrete Distributions

1. Find the probability that in five tosses of a fair die a 3 appears:
   a. At no time.
   b. Once.
   c. At least two times.

2. Let X be r.v. having the Binomial distribution with parameters n, p such that $E(X) = 10$ and $\text{var}(\text{X}) = 6$ find n and p.

3. If X has the binomial distribution with $\mu = 3.2$ and $\sigma_X = 0.8$ find n and p.

4. If X has the binomial distribution with $\mu = 2$ and ${\sigma^2}_X = 1.2$ find n and p

5. If X has the binomial distribution with n=100 and p=0.1, Find $P(X \leq E(X) - 3Var\ (X))$

6. The probability that a certain kind of component will survive a given shock test is 0.6 find the probability that out of 7 components tested, at most 5 survive.

7. A long time ago the occupational disease in an industry was such that the workmen had a 20% chance of suffering from it. If seven workmen were selected at random, find the probability that at most 5 of them contracted the disease.

8. An automobile safety engineer claims that 1 in 10 automobile accidents is due to driver fatigue. What is the probability that at least 2 of 5 automobile accidents are due to driver fatigue?

9. The probability that a patient recovers from a rare blood disease is 0.6. If 7 people are known to have contracted this disease what is the probability that:
   a. Exactly 4 survive. b. At least 5 survive.

10. The probability that a certain kind of vacuum tube will survive a thermal shock test is 0.85. Find the probability that among 20 such tubes
    a. Exactly 17 will survive.
    b. At least 15 will survive.

11. The probability that a college student doesn't graduate is 0.3. Five college students are chosen at random, find the probability that:
    a. Three will not graduate.
    b. At least one will not graduate.

12. Suppose that for a very large shipment of integrated circuit chips. The probability of failure for any one chip is 0.10. Find the probability that at most 3 chips fail in a random sample of 20.

13. A Basketball player hits on 80 % of his shots from the free throw line. What is the probability that he makes at most 4 of his next 5 free shots?

14. If 40 % of the fuses produced by a company are defective what is the probability that at least 2 out of 5 fuses chosen at random will be **non defective**.

15. A manufacturer knows that on the average 20% of the electric toasters which he makes will require repairs within 1 year after they are sold when 20 toasters are randomly selected, find appropriate numbers x and y such that:

a. The probability that at least x of them will require $\leq \frac{1}{2}$

b. The probability that at least y of them will not require is $\geq 1/2$.

16. When calculating all the values of a binomial distribution, the work can usually be simplified by first calculating $b(0; n, p)$ and then using the recursion formula

$$b(x+1; n, p) = \frac{(n-x)p}{(x+1)q} . b(x; n, p).$$

Verify this formula and use it to calculate the values of the binomial distribution with $n = 5$ and $\mathrm{p} = 0.25$.

17. A box of fuses contain 40 fuses, of which 8 are defective. If 5 of the fuses are selected at random and removed from the box in succession **without replacement**, what is the probability that at most two fuses will be defective?

18. A box contains 100 microchips, 80 good and 20 defective. The number of defectives in the box is unknown to a purchaser, who decides to select 10 microchips at random **without replacement** and to consider the box acceptable if the 10 items selected include no more than 3 defectives. Find the probability of accepting this box.

19. Prove the following recursion formula for the hypergeometric distribution,

$$h(x+1; n, M, N) = \frac{(n-x)(M-x)}{(x+1)(N-M-n+x+1)} . h(x; n, M, N)$$

and use it to calculate the values of the hyper geometric distribution with $\mathrm{n} = 4$,$N = 9$ and $M = 5$.

20. In a “torture test” a light switch is turned on and off until it fails. If the probability is 0.001 that the switch will fail any time it is turned on or off, what is the probability that the switch will not fail during the first 800 times it is turned on or off? Assuming that the conditions underlying the geometric distribution are met.

21. If the probability is 0.75 that a person will belive a rumor about the transgressions of a certain politician, find the probabilities that:

a. The fifth person to hear the rumor will be the first to believe it.

b. The eighth person to hear the rumor will be the third to believe it.

22. If X has the geometric distribution with $P(X = 1) = 4P(X = 2)$, Find $P(X > 1)$.

23. If X has the geometric distribution with $P(X = 1) = 0.4$, Find $P(X > 2)$.

24. If X is a r.v. having a geometric distribution, show that

$$P(X = m + n/X > n) = P(X = m)$$

25. Differentiating w.r.t p the expressions on both sides of the equation

$$\sum_{x=1}^{\infty} p(1-p)^{x-1} = 1$$

show that the mean of geometric distribution is given by $\mu = \frac{1}{p}$.

26. If X and Y have geometric and Poisson distribution respectively, $E[Y] = 2\ln E[X]$ and $P(X = 2) = P(Y = 0)$ Find $P(Y > 0)$

27. If the r.v. X has the Poisson distribution with $P(X = 1) = 2P(X = 2)$, Find $P(X > 2)$.

28. If X has a Poisson distribution $P(X = 0) = 1/2$ what is μ_X , σ_X^2.

29. If X has the Poisson distribution with $P(X = 0) = 2P(X = 1)$ Find $P(X > 1)$.

30. If X has the Poisson distribution such that $P(X = 2) = 3P(X = 1)$ Find the mean.

31. If X has a Poisson distribution with parameter λ such that $P(X = 2) = 3\,P(X = 4)$ find E(X).

32. Suppose 2% of people on the average are left handed. Find the probability that at least four are left handed among 200 people.

33. If the probability that an individual suffers a bad reaction from the injection of a given serum is 0.001, determine the probability that out of 2000 individuals exactly 5 will suffer a bad reaction.

34. Suppose that 1% of all transistors produced by a certain company are defective. A new model of computers requires 100 of these transistors, and 100 are selected at random from the company's assembly line. Find the probability of obtaining 3 defectives. [Ans = 0.0613]

35. If 2% of the books bound at a certain bindery have defective bindings, use the Poisson approximation to the binomial distribution to determine the probability that five of 400 books bounded by this bindery will have defective bindings. [Ans = 0.093]

36. Telephone calls enter a college switch board at a mean rate 1/3 call per minute according to a Poisson process. Find the probability that at most 10 telephone calls will enter the college switch board during two hours.

37. The number of monthly breakdown of a computer is a r.v. having a Poisson distribution with $\lambda = 1.8$. Find the probabilities that this computer will function a month:

a. Without a breakdown; **b.** With only one breakdown

38. The average number of radioactive particles passing through a counter during 1 millisecond in a laboratory experiment is 4. What is the probability that 6 particles enter the counter in a given millisecond? What is the probability that at least 10 particles enter the counter in 2.5 milliseconds?

39. The average number of trucks arriving on any one day at a truck depot in a certain city is known to be 4. What is the probability that on a given day at least three trucks will arrive at this depot?. Also what is the probability that on a period of 3 days between 12 and 15 trucks will arrive at that depot? [Ans = 0.7619 ; 0.3829]

40. A certain kind of sheet metal has, on average, five defects per 10 square feet. If we assume a Poisson distribution, what is the probability that a 15 square feet of metal will have at least 6 defects? [Ans. = 0.7586]

Continuous Distributions

41. In certain experiments, the error made in determining the density of a substance is a r.v. having a uniform density with $\alpha = -0.015$ and $\beta = 0.015$ .Find the probabilities that an error will

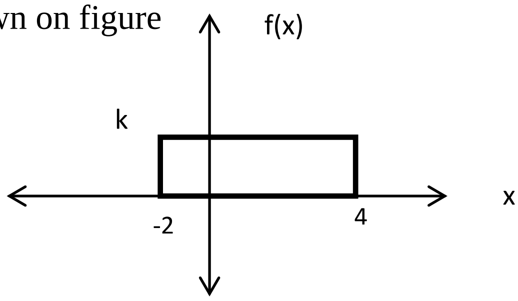
a. be between -0.002 and -0.003 .

b. exceed 0.005 in absolute value.

42. If X has a uniform distribution with p.d.f. $f(x) = \frac{1}{7}, k < x < 17$ find the value of k.

43. If X has a uniform distribution over (1,2) and P (X < $\mu + c$) = 0.2 , find the value of c.

44. If X has a uniform distribution with a p.d.f. as shown on figure
Find:



a. Value of k
b. $P(X > 0)$
c. $P(-1 < X < 3)$
d. $P(X < 2)$

45. If the r.v. X has the exponential distribution, prove that

$$P(X > \alpha + \beta \ / \ X > \ \alpha) = P(X > \beta)$$

where α and β are any positive constants.

46. If X has exponential distribution with mean =1.Find $P(X > \ 1 \ / \ X > 2)$

47. If X has exponential distribution with standard deviation = 2. Find P(X< 1 / X<2).

48. Assume the length x in minutes of a particular type of telephone conversation is a r.v. with p.d.f

$$f(x) = \begin{cases} \frac{1}{5} e^{-\frac{x}{5}} & for\ x > 0 \\ 0 & otherwise \end{cases}$$

a. Determine the mean length $E(X)$ of this type of telephone conversation.
b. Find the variance
c. S.D of X.
d. Find $E(X + 5)^2$.

49. Suppose it is known that the life X of a particular compressor in hours has the p.d.f.

$$f(x) = \begin{cases} \frac{1}{900} e^{-\frac{x}{900}} & for\ x > 0 \\ 0 & otherwise \end{cases}$$

a. Determine the mean life of the compressor.
b. Find the variance and S.D of X.

50. Suppose a certain solid state component has a life time or failure time (in hours) X ~ EXP(100). Find the probability that the component will last at least 80 hours given that it already worked more than 30 hours. [Ans. = 0.6065]

51. If $Z \sim N(0,1)$ and $P(-A \leq Z \leq A) = 0.4$ Find $P(Z > A)$.

52. Given the normally distribution r.v. X with $\sigma_x^2 = 25$ and $P(X < 35) = 0.719$ .Find μ.

53. If $X \sim N(0,1)$ and $P(0 \leq Z \leq c) = 0.3$ Find $P(Z > c)$.

54. Given the normally distribution r.v. X with $\mu = 25$ and $P(X > 10) = 0.9332$.Find σ_x

55. Given $X \sim N(\mu, \sigma_x^2)$ with $\sigma_x^2 = 25$ and $P(X \leq 45) = 0.8849$ Find μ.

56. If X~N (400,2500), find P (360 < X < 469) and P(X = .400),

57. If X~ N(0,9).find P($|X|$< 1)

58. The lengths of components produced by a machine are normally distributed with standard deviation 1.5 mm. At a certain setting 25% of the components are longer than 100 mm. Calculate the mean value of the lengths at this setting?

59. If the heights of 500 students are normally distributed with mean 70 inches and variance 16 inch^2. How many students you expect to have heights between 66 and 72 inches?

60. An electrical firm manufactures light bulbs that have a length of life that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. [Ans. = 0.511].

61. The speeds of motorists passing a particular point on a motorway are found to be normally distributed with mean 115 km/h and standard deviation 8 km/h.

a. Find the percentage of motorists whose speeds exceed 120 km.
b. Find v such that the speed of 20% of the motorists does not exceed v km/h.

62. The volume (in liters) of liquid in bottles filled by a machine is normally distributed with mean 1.02 and standard deviation 0.01 liter,

a. What is the probability that a bottle, selected at random, contains less than 1 liter.
b. To what value must the mean be altered to reduce the probability (a) to 1% (assuming the standard deviation is unaltered)?

63. A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy what percentage of resistors will have a resistance that exceeds 43 ohms? [Ans. = 6.68%]

64. Gauges are used to reject all components in which a certain dimension is not within the specification $1.50 \pm d$. It is known that this measurement is normally distributed with mean 1.50 and standard deviation 0.2. Determine the value d such that the specifications "cover" 95% of the measurements. [Ans. = 0.392].

65. A manufacturer's process for producing coke cans can be regulated so as to produce cans with an average weight 200 gm, if the weights are normally distributed with variance 100 gm. Find the percentage of cans such that:

a. The weight of a can in more than 215 gm

b. The weight of can between 205 gm and 215 gm

66. A controlled satellite is known to have an error (distance from target) that is normally distributed with mean zero and standard deviation 4 feet. The manufacturer of the satellite defines a "success" as a firing in which the satellite comes within 10 feet of the target. Compute the probability that the satellite fails.

CHAPTER 5: JOINT DISTRIBUTIONS

Some useful rules

Discrete Random Variables

Joint P.m.f.

$h(x, y) = P(X = x, Y = y)$

where

$h(x, y) \geq 0$ and $\sum_x \sum_y f(x, y) = 1.$

Marginal Distribution

$$g_X(x) = \sum_y h(x, y)$$

$$g_Y(y) = \sum_x h(x, y)$$

Conditional density function

$$f_{X/Y}\ (x_i/y_j) \ = \ \frac{h(x_i, y_j)}{g_Y(y_j)}$$

$$f_{Y/X}\ (y_j/x_i) = \frac{h(x_i, y_j)}{g_X(x_i)}$$

Continuous Random Variables

Joint P.d.f.

$$\iint_A f(x, y)\, dx\, dy = P((x, y) \in A)$$

where

$f(x, y) \geq 0$ and $\iint_{-\infty}^{\infty} f(x, y) = 1.$

Marginal Distribution

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy \quad , -\infty < x < \infty$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y)\, dx \ -\infty < y < \infty$$

Conditional density function

$$f_{X/Y}(x/y) = \frac{f(x,y)}{f_Y(y)} \qquad f_{Y/X}\ (y/x) \ = \frac{f(x,y)}{f_X(x)}$$

Expectation

$$E(XY) = \begin{cases} \displaystyle\sum_x \sum_y x\ y\ f(x, y) & \text{, if X and Y are descrete} \\ \displaystyle\iint_{-\infty}^{\infty} x\ y f(x, y) dx\ dy & \text{, if X and Y are continous} \end{cases}$$

Covariance

$$\sigma_{XY} = \text{Cov}(X, Y) = \ E(XY) - \ \mu_X\ \mu_Y$$

Correlation coefficient

$$\rho(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var(X)\, Var(Y)}} \ = \ \frac{\sigma_{XY}}{\sigma_X \ \ \sigma_Y}$$

Exercises

Discrete joint distribution

1. A pair of fair dice is thrown, let X assign the maximum of the two numbers which appear and Y the absolute difference. Find the joint probability function of X and Y then calculate the correlation coefficient between X and Y.

2. If the joint p.m.f. of X and Y is given by
$$h(x\,,y) = \,c\,(x^2\, +\, y^2) \quad ,\, x\, =\, -1, 0, 1{,}3 \text{ and }\, y\, =\, -1{,}2{,}3.$$
Find the constant c, the marginal distributions of X and Y, $COV(X,Y)$ and $\rho(X,Y)$

3. The joint probability function of two discrete random variables X and Y is given by
$$f(x,y) = c\,x\,y \;\; , \text{ for } x \,=\, 1{,}2{,}3, \;\text{ and } y \,=\, 1{,}2{,}3.$$
Find:
a. The constant c.
b. $P(X = 2\,, Y > 2)$, then $P(X > 2\,)$.
c. the marginal probability functions of X and that of Y
d. the conditional probability of Y given X.
e. the conditional probability of X givenY.

4. Suppose that X and Y are two discrete r. v.'s with joint density function given in the following table:

X / Y	1	2	3
1	0	1/6	1/12
2	1/5	1/9	0
3	2/15	1/4	1/18

a. Find the marginal probabilities of X and Y.
b. Find the conditional density $f_{X/Y}$

5. If the joint probability distribution of X and Y is given by the following table

Y / X	-2	-1	2	4
1	0.1	0.2	0.0	0.1
2	0.0	0.1	0.1	0.2
3	0.1,	0.0	0.1	0.0

Find:$P(Y > X)$, $COV(X,Y)$ and $Var(2X - 5Y + 7)$.

Continuous joint distribution

6. Determine the constant k so that

$$f(x,y) = \begin{cases} kx(x-y) & ; \quad 0 < x < 1\,, |y| < x \\ 0 & ; \quad O.W. \end{cases}$$

can serve as a joint probability density function. Then find, $P(0 < Y < X < 1/2)$ and $P(X - Y > 1)$

7. if X and Y have the joint probability density function given by:

$$f(x\,,y) = \begin{cases} k & ; \quad 0 < y < x < 2 \\ 0 & ; \quad o.w. \end{cases} .$$

Find:

a. The value of k,

b. $P(X - Y > 1)$ and $P(Y > 1/X > 1)$,

c. $\mathrm{Cov}(3X - 7{,}2Y + 3)$

11. If the joint p.d.f. of X and Y is given by

$$f(x\,,y) = \begin{cases} k\,x\,y & ; \quad 0 < y < x < 1 \\ 0 & ; \quad o.w. \end{cases}$$

a. Find the value of k.

b. Show that X and $Y$are dependent.

c. Calculate $\mathrm{Cov}(X, Y)$

8. Let X and Y be continuous random variables having joint density function: $f\ (x, y\ ) = \begin{cases} k\ (x^2 + y^2)\ ; & 0 \le x \le 1,\ 0 \le Y \le 1 \\ 0 & o.w \end{cases}$

Find

a. The constant k

b. $P(\mathrm{X} < 1/2\ ,\ Y > 1/2\ )$

c. The marginal distribution function of X and of Y,

d. then determine whether X and Y are independent.

9. Let $f(x,y) = \begin{cases} e^{-(x+y)}\ ; & x \ge 0, \quad y \ge 0 \\ 0 & o.w \end{cases}$ be a joint density function of X and Y, find the conditional density function of:

a. X given Y

b. Y given X

10. Two random variables X, Y have the joint density function given by

$$f(x,y) = \begin{cases} 4xy\,; & 0 < x < 1\,, 0 < y < 1 \\ 0 & o.w \end{cases}$$

- **a.** Find $P(0 \leq X \leq 3/4\ ,\ 1/8 \leq Y \leq 1/2)$
- **b.** Find $P\ (\ X > \ 1/2)$
- **c.** Find the expected value of $Z\ =\ x^2 + y^2$
- **d.** The marginal distribution functions of X.

11. Two random variables X, Y have joint density given by:

$$f\ (x,y) = \begin{cases} k(x^2 + y^2) & 0 < x < 2, \quad 1 < y < 4 \\ 0 & o.w \end{cases}$$

- **a.** Find the value of k
- **b.** Find $P\ (1 < X < 2\ ,\ 2 < Y \leq 3)$
- **c.** Find the probability that $X + \mathrm{Y} > 4$
- **d.** Are X and Y independent?

12. If X and Y have the joint density function

$$f(x,y) = \begin{cases} e^{-(x+y)} & ; x > 0, y > 0 \\ 0 & o.w. \end{cases}$$

If X and Y are independent, find $P(0 < X < 1\ /\ Y = 2\ )$

13. If Var(X_I)= 5, Var(X_2)=4, Var(X_3)=7, COV(X_1X_2)=3, COV(X_1,X_3)=-2, and X_2 and X_3 are independent. Find the correlation coefficient of Y = X_I - 2X_2 + 3X_3 and Z = -2X_1 + 3X_2 + 4X_3.

Appendix A : Normal Distribution Table

Probabilities for the standard normal distribution

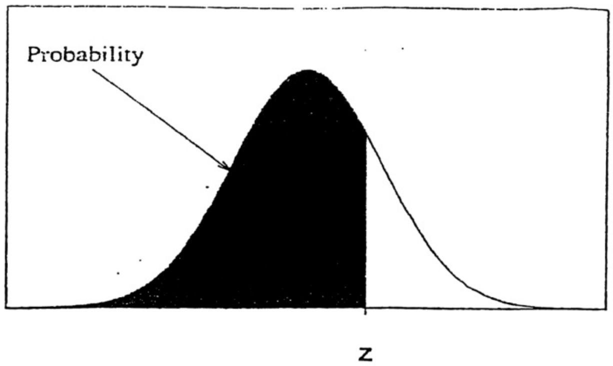


Table entry for z is the probability lying to the left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Appendix B

Techniques of Counting

Multiplication rule:

Suppose a Job consists of k stages, and the i^{th} stage can be carried out in n_i ways, irrespective of how the other stages are carried out. Then the whole job can be carried out in $n_1.n_2 \ldots\ldots n_k$ ways.

Addition rule:

If an event A_1 can be done in n_1 ways and an event A_2 can be done in n_2 ways and $A_1$and A_2 are mutually exclusive, then the number of ways these events can occur is $n_1 + n_2$.
In general, the number of ways k mutually exclusive events can occur is $n_1 + n_2 + \ldots + n_k$.

Sampling :

There are 4 ways of sampling k object from a group of n distinct objects based on order and repetition

	order	Without order
With repetition	n^r	${}^{n+k-1}C_k$
Without repetition	${}^nP_k = \dfrac{n!}{(n-k)!}$	${}^nC_k = \dfrac{n!}{k!\quad(n-k)!}$

Permutations with repetition

Suppose that we are given n object of which n_1 are similar, n_2 are similar, … , n_k are similar. Then the number of different permutations of these objects is $\frac{n!}{n_1!\ n_2!\ \ldots\ n_k!}$

Partitions

Given a group A of n distinct objects, then there exists different ordered partitions of A of the form $A_1, A_2, \ldots , A_k$. such that A_i contains n_i objects, $i = 1,2,3, \ldots , k$.
where $n_1 + n_2 + \cdots + n_k = n$

a) **Ordered partitions**:

If the order between the subgroups matters then
The number of these partitions = $\frac{n!}{n_1!\ n_2!\ \ldots\ n_k!}$ which is equivalent to ${}^nC_{n_1}\ {}^{n-n_1}C_{n_2}\ \ldots\ldots\ {}^{n-n_1-\ldots-n_{k-1}}C_{n_k}$

b) **unordered partitions**:

If the order between the k subgroups **does not** matters then
The number of these partitions = $\frac{\#\text{ of ordered partations}}{k!}$

Example: In how many ways can 14 students be partitioned into 6 teams, 2 of them contain 3 students each and the others contain 2 students each.

Answer: since there is no difference between the teams containing the same under of students then this is a case of unordered partitions and the # of ways = $\frac{14!}{(2!)^6\ (3!)\text{^}2} \quad \frac{1}{2!\ \ 4!}$

Pigeonhole Principle

- If $k + 1$ or more objects are placed into k boxes, then there is at least one box containing two or more objects.
- If N objects are placed into k boxes then there is at least one box containing at least $\left\lceil \frac{N}{k} \right\rceil$

Useful formulas:

a) ${}^{n}C_k = {}^{n}C_{n-k}$

b) ${}^{n+1}C_k = {}^{n}C_k + {}^{n}C_{k-1}$

c) $k\ {}^{n}C_k = n\ {}^{n-1}C_{k-1}$

d) $\sum_{k=0}^{n} {}^{n}C_k = 2^n$

e) $\sum_{k=0}^{n} (-1)^n\ {}^{n}C_k = 0 \quad , n > 0$

f) ${}^{k}C_k + {}^{k+1}C_k + \ldots {}^{k+n}C_k = {}^{n+k+1}C_{k+1}$

APPINDIX C: Final Answer to Sheet Problems

CHAPTER 2: PROBABILITY

- ***Probability Concepts***

1.

a. $A\bar{B}\bar{C}$

b. ABC

c. $\bar{A}\bar{B}\bar{C}$

d. $A \cup B \cup C$

e. $\overline{ABC}$

f. $\bar{A}BC \cup A\bar{B}C \cup AB\bar{C} \cup ABC$

g. $A\bar{B}\bar{C} \cup \bar{A}B\bar{C} \cup \bar{A}\bar{B}C$

2. Set C consists of the citizens of a certain town who voted "YES" for water fluoridation. Set D of consists of the citizens of the same town who have preschool children. Define:

- **a.** Set of citizens of a certain town who voted "YES" for water fluoridation and do not have preschool children
- **b.** Set of citizens of a certain town who **did not** vote "YES" for water fluoridation and have preschool children
- **c.** Set of citizens of a certain town who **did not** vote "YES" for water fluoridation, OR do not have preschool children

3. Ans. = 46

4.

a. $P(A_1) = P(A_2) = P(A_3) = 1/3$.

b. $P(A_1) = P(A_2) = 1/4$.

c. $P(A_1) = \frac{6}{11},\ P(A_2) = \frac{3}{11},\ P(A_3) = \frac{2}{11}$.

5.

a. P(pass) + P(not pass) must equal 1. (there is no error in this statement)

b. $P(A \cup B) \leq P(A) + P(B)$ (not bigger 0.95> 0.77+0.08)

c. Sum of these probabilities should equal one. (not 1.03)

d. Sum of these probabilities should equal one (not 0.98)

6.

a. $P(e_1) = \frac{1}{3}, P(e_2) = \frac{1}{6}$. **b.** $P(e_1) = \frac{1}{6}$

7. P(d)=0.38 and P(e)=0.19.

8.

a. 0.29 b.0.8

9.

a. 0.46 b. 0.40 c. 0.11 d. 0.68

10. P (G) = 4/9.

11.

a. 1/5. b. 1/3.

12.

a. 0.88, b. 0.12, c. 0.34.

13. 3/4

14. 2/3

15.

a. 0.1 **b.** 17/150.

16.

a. 0.15. b. 0.50. c. 0.70. d. 0.65.

17.

a. $P(\bar{A}) = 0.63$ **d**. $P(\overline{B}) = 0.66$

b. $P(A \cup B) = 0.81$ **e**. $P(A \cap \overline{B}) = 0.37$

c. $P(\bar{A} \cap B) = 0.44$ **f**. $P(\overline{A} \cap \overline{B}) = 0.19$

18.

a. 3/5 b. 3/5

19.

a. 0.21 b. 0.01

- ***<u>Conditional probability</u>***

20.

a. 1/9 **b.** 2/9

21.

c. Hint: $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

22.

a. $P(G) = 0.60$ **b**. $P(\overline{T}) = 0.70$ **c**. $P(G \cap T) = 0.20$

d. $P(\overline{G} \cap T) = 0.10$ **e**. $P(T/G) = 18/54$ **f**. $P(\overline{G}/T) = 9/27$

23. Hint: $\frac{P(C\cap A\cap B)}{P(A\cap B)} = \frac{P(B\cap C)}{P(B)}$, divide both sides by $P(B \cap C)$ then multiple by $P(A \cap B)$.

24. Hint: $P(A \cap B) = P(A)P(B)$.

25. P(A/B) = 0.75, P(A/$\bar{B}$)= 3/8, P($\bar{A}$/B) = 0.25 and P($\bar{A}$/$\bar{B}$) = 5/8.

26.

a. Zero **b.** one **c.** $\frac{P(B)}{P(A)}$

27. 0.625

28. $\left(\frac{5}{20}\right)\left(\frac{4}{19}\right)\left(\frac{3}{18}\right) = \frac{1}{114}$

29.

a. $14/15$ **b.** $1/3$ **c.** 0.20

30.

a. 0.625 **b.** 0.50 **c.** 0.10

31.

a. 78/83 **b.** 78/82

32. 11/25 = 0.44

33. 0.32.

34. 23/50 = 0.46.

35. Assuming that n ≤ 365.

a) $1 - \frac{(365)(364)(363)\ldots.(365-n+1)}{(365)^n}$ b) $\cong 0.97$. c) n = 23 , Pr $\cong 0.507$

- ***<u>Independent events</u>***

36. Hint: to prove 2 event are independent, you have to prove $P(A \cap B) = P(A)P(B)$

37.

a. 1/12. **b.** 1/9. **c.** 1/3.

38. P(A) = P(B) = 0.4.

39.

a. 1/12. **b.** 1/9.

40.

a. 47/60. **b.** 36/60 = 0.6.

41. A ={HHT, HHH} , B={HHT, HTT, THT, TTT}, C = {HTT, THT, TTH}

a. $\because P(A \cap B) = \frac{1}{8} = P(A)P(B)$. **b.** $\because P(B \cap C) = \frac{3}{16} \neq P(B)P(C)$. .

42.

a. $\left(\frac{5}{8}\right)\left(\frac{5}{8}\right) + \left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = \frac{17}{32}$ **b.** $\left(\frac{5}{8}\right)\left(\frac{3}{8}\right) + \left(\frac{3}{8}\right)\left(\frac{5}{8}\right) = \frac{15}{32}$

43.

a. $\left(\frac{5}{8}\right)\left(\frac{4}{7}\right) + \left(\frac{3}{8}\right)\left(\frac{2}{7}\right) = \frac{13}{28}$ **b.** $\left(\frac{5}{8}\right)\left(\frac{3}{7}\right) + \left(\frac{3}{8}\right)\left(\frac{5}{7}\right) = \frac{15}{28}$

44.

a. $\left(\frac{8}{20}\right)\left(\frac{7}{19}\right)\left(\frac{6}{18}\right)$,

b. $\left(\frac{3}{20}\right)\left(\frac{2}{19}\right)\left(\frac{1}{18}\right)$,

c. $6\left(\frac{8}{20}\right)\left(\frac{3}{19}\right)\left(\frac{9}{18}\right)$.

d. $3\left(\frac{8}{20}\right)\left(\frac{7}{19}\right)\left(\frac{9}{18}\right)$,

e. $= \left(\frac{23}{57}\right)$

45.

a. 43/96. **b.** 10/43

46. 1 – P(no double 6 occurs) = $1 - \left(\frac{35}{36}\right)^{10} = 0.2455$.

47.

a. $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$, **b.** $3\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \frac{5}{72}$.

c. $\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{5}{36}$ **d.** $1 - \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{19}{27}$.

48. 0.588

49. 0.13456

50. 0.776953125

51. 11/16 = 0.6875

- ***<u>Total probability and Bayes' formula</u>***

52. $(0.60)(0.09) + (0.30)(0.20) + (0.10)(0.06) = 0.12$

53. B_i = the event of i successful shoots. $P(B_0)=0.09$, $P(B_1)=0.36$, $P(B_2)=0.41$, $P(B_3)=0.14$, P(destroyed) = 0.458 , $P(B_1 / D) = 0.1572$

54. $(0.40)(0.84) + (0.6)(0.49) = 0.63$

55. P(defective) = 1/32 , P(Y/D) = 0.40.

56. P(defective) = 23/66 , $P(B_3/D) = 6/23$.

57. P(defective) = 0.039, P(C/D) = 4/13.

58. P(defective) = 0.0455 , P(B/D) = 18/91.

59. P(B) = 0.005985, P(A/B) =22/133 $\cong 0.1654$

60.

a. $P(y_0) = 0.55$, $P(y_1) = 0.45$.
b. $P(x_0/y_0) = 0.818$ **c.** $P(x_1/y_1) = 0.889$ **d.** $P_e = 0.15$.

- ***<u>Techniques of counting (Enumeration methods)</u>***

61. ${}^{7}P_3 = 210$

62. $\frac{11!}{1!\ 4!\ 4!\ 2!} = 3450$

63.

a. $9.10.10.10 = 9000$. **b.** 9. 9. 8. 7 = 4536.
c. 9.8.7 = 504 . **d.** 8. 8. 7. 4 + 504 = 2296

64.

a. 2 4! 4! = 1152 **b.** 5! 4! = 2880

65. 5! 4!

66.

a. 4! 6! 2! 3! = 207360, **b.** 9! 4!

67. ${}^{5}C_3\ {}^{5}C_4 + {}^{5}C_4\ {}^{5}C_3 + {}^{5}C_5\ {}^{5}C_2 = 110$

68. $\frac{10!}{5!\ 2!\ 3!} = 2520$

69.

a. ${}^{10}C_7 = 120$ **b.** ${}^{10}C_5\ {}^{5}C_3 = 2520$. c. **.** ${}^{10}C_5 \div 2$

70.

a. ${}^{5}C_2 \;\; {}^{7}C_3.$ **b.** ${}^{5}C_2 \;\; {}^{6}C_2.$ **c.** ${}^{3}C_2 \;\; {}^{7}C_3.$

71. ${}^{8}C_3 = 56$

72. 16/52 = 4/13.

73.

a. $\frac{{}^{13}C_2 \;\; {}^{4}C_2 \;\; {}^{4}C_2 \;\; {}^{44}C_1}{{}^{52}C_5}$ **b.** $\frac{{}^{13}C_1 \;\; {}^{4}C_4 \;\; {}^{48}C_1}{{}^{52}C_5}$ or $\frac{13 \;\; . \;\; 48}{{}^{52}C_5}$

74. $\frac{{}^{110}C_3}{{}^{120}C_3} \cong 0.7685$

75. $\frac{{}^{5}C_2 \;\; {}^{10}C_2 \;\; {}^{12}C_2 \;\; {}^{3}C_2}{{}^{30}C_8}$

76. 5! ${}^{5}C_3 \;\; {}^{4}C_2$

77.

a. $\frac{{}^{3}C_1 \;\; {}^{47}C_9}{{}^{50}C_{10}}$ **b.** $1 - \frac{{}^{3}C_3 \;\; {}^{47}C_7}{{}^{50}C_{10}}$

<u>Circle the correct answer from each of the following multiple choice questions:</u>

1- C) 0.4
2- D)0.96
3- B) Independent
4- D) mutually exclusive
5- C) $A \subset B$
6- A) independent
7- B) $B \subset A$
8- C) 0.1

<u>Against each statement, put a tick (√) if it is TRUE and a (×) if it is FALSE:</u>

1- False
2- True
3- True
4- False
5- False
6- True

CHAPTER 3: RANDOM VARIABLES

Discrete Random Variable

1.

a. $f(1) = \frac{-1}{5}$, not a pmf **b.** is a pmf. **c.** $\sum f(x) = \frac{6}{5} \neq 1$, not a pmf.

2.

a. $\frac{1}{5}$ **b.** $\frac{1}{32}$ **c.** 3 **d.** $\pm\frac{1}{\sqrt{n}}$

3.

a. No since F(4)= 1.2 > 1 **b.** No, since F(1) > F(2) **c.** Yes.

4. $|k| < 1$

5.

a. 0.5

b. 0.25.

c.

x	-1	1	3	5
$f(x)$	¼	¼	¼	¼

6.

a.

x	1	4	6	10
$f(x)$	1/3	1/6	1/3	1/6

b. $\frac{1}{6}$ **c.** $\frac{1}{6}$ **d.** Mean = 14/3, σ^2 =89/9 , σ =3.14466 **e.** 1424/9

7.

a.

x	-2	-1	0	1	2
$f(x)$	1/9	2/9	3/9	2/9	1/9

b. $\mu = -1$, σ^2 =16/3. **c.** 1/3, 5/9

8.

a. 2/25

b. $F(x) = \begin{cases} 0 & x < -3 \\ \frac{1}{25} & -3 \le x < -2 \\ \frac{3}{25} & -2 \le x < -1 \\ \frac{7}{25} & -1 \le x < 0 \\ \frac{18}{25} & 0 \le x < 1 \\ \frac{22}{25} & 1 \le x < 2 \\ \frac{24}{25} & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$

c. $E(X+4)$ =4 , Var(-2X+1)=168/25 **d.** 21/25

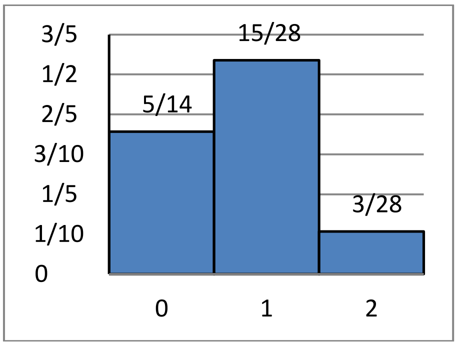
9. a = 1 , b = 1/3. **b.** 4/3

c. $F(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{9} & -2 \le x < -1 \\ \frac{3}{9} & -1 \le x < 0 \\ \frac{6}{9} & 0 \le x < 1 \\ \frac{8}{9} & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$

d. 6/9 , 7/9

10.

x	0	1	2
$f(x)$	10/28	15/28	3/28



11.

x	0	1	2	3
$f(x)$	1/27	6/27	12/27	8/27

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{27} & 0 \le x < 1 \\ \frac{7}{27} & 1 \le x < 2 \\ \frac{19}{27} & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

P(1<X≤3) = 20/27,   P(X>2) = 8/27.

y	-3	-1	1	3
$f(y)$	1/27	6/27	12/27	8/27

12.

a. $\mu = n\ ,\ \sigma^2 = n(n-1)$

b. $\mu = \frac{n+1}{2},\ \sigma^2 = \frac{n^2-1}{12}$

Hint: **a.** $\sum_{x=0}^{\infty} q^x = \frac{1}{1-q}$; $|q| < 1$ by diff. w.r.t q $\sum_{x=1}^{\infty} x\, q^{x-1} = \frac{1}{(1-q)^2}$; $|q| < 1$

Similarly : $\sum_{x=1}^{\infty} x^2\, q^{x-2} = \frac{1+q}{(1-q)^3}$; $|q| < 1$

b. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2},\ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

13.

x	0	1	2
$f(x)$	12/22	9/22	1/22

Mean = 0.5

Continuous Random Variable

14.

a. c = 2

b. $F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 < x < 1 \\ 2x - \frac{x^2}{2} - 1 & 1 < x < 2 \\ 1 & x > 2 \end{cases}$

c. $P(0.8 < X < 1.2) = 0.36$

d. mean = 1 , variance = 1/6

15.

a. $k = \frac{1}{2}$,

b. $\boldsymbol{F(x)} = \begin{cases} \mathbf{0} & \boldsymbol{x} < 0 \\ \frac{x^2}{2} & \mathbf{0} < x < 1 \\ \frac{x}{2} - \frac{1}{4} & \mathbf{1} < x < 2 \\ \frac{3x}{2} - \frac{x^2}{4} - \frac{5}{4} & \mathbf{2} < x < 3 \\ \mathbf{1} & \boldsymbol{x} > 3 \end{cases}$

c. $P(0.8 < X < 1.2)$ = F(1.2) - F(0.8) = 0.19, using the p.d.f. $\int_{0.8}^{1.2} f(x)dx = 0.19$.

16.

a. $\boldsymbol{f(x)} = \begin{cases} \frac{3x^2}{2} & -\mathbf{1} < x < 1 \\ \mathbf{0} & \textbf{otherwise} \end{cases}$

b. $P(-1.5 < X < 1) = 1$

c. E(X)=0 and Var(X)= 0.6

17.

a. $1 - 3e^{-2}$; **b.** $4\,e^{-3}$

18.

a. $P(X < 2) = 1 - 3\,e^{-2}$, $P(1 < X < 3) = 2\,e^{-1} - 4\,e^{-3}$, $P(X > 4) = 5\,e^{-4}$

b. $f(x) = \begin{cases} x\,e^{-x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$

19.

a. k = 12

b. $F(x) = \begin{cases} 0 & x < 0 \\ 6x^2 - 8x^3 + 3x^4 & 0 < x < 1 \\ 1 & x > 1 \end{cases}$

c. E (X)= 0.4 and Var (2 – 3 X) = 0.36 **d.** $P(0.3 < X < 1.2) = 0.6517$

20.

a. k = 12

b. $F(x) = \begin{cases} 0 & x < 0 \\ 4x^3 - 3x^4 & 0 < x < 1 \\ 1 & x > 1 \end{cases}$

c. E (X) = 0.6 and Var(3–4X) = 0.64 **d.** P (-1 < X < 0.7) = 0.6517

21.

a. Hint : $f(1) \geq 0$, $\int_0^1 f(x)dx = 1$

b. E (X)= 8/15, $\text{Var}(2X - 3) = \frac{44}{225} = 0.19556$

c. $P(0.2 < X < 0.8) = \frac{99}{125} = 0.792$.

22.

a. k = 0.3

b. $\boldsymbol{F(x)} = \begin{cases} \mathbf{0} & \boldsymbol{x} < 0 \\ \mathbf{0.1}\boldsymbol{x}^{\mathbf{3}} & \mathbf{0} < x < 1 \\ \mathbf{0.15}\boldsymbol{x}^{\mathbf{2}} - \mathbf{0.05} & \mathbf{1} < x < 2 \\ \mathbf{1.2}\boldsymbol{x} - \mathbf{0.15}\boldsymbol{x}^{\mathbf{2}} - \mathbf{1.25} & \mathbf{2} < x < 3 \\ \mathbf{1} & \boldsymbol{x} > 3 \end{cases}$

c. E (X) = 15/8 , Var (1- 4X) = 6.31 , P (1.8 < X < 3.2) = 0.564

23.

a. k = ¾ **b.**

c. E (X) =0 **d.** Var (1-5 X) = 5

e. P (-1.5 < X < 0.5) = 27/32

b. $F(x) = \begin{cases} 0 & x < -1 \\ \frac{3}{4}x - \frac{1}{4}x^3 + \frac{1}{2} & -1 < x < 1 \\ 1 & x > 1 \end{cases}$

24.

a. $k = 6$

b. $F(x) = \begin{cases} 0 & x < 0 \\ 3x^2 - 2x^3 & 0 < x < 1 \\ 1 & x > 1 \end{cases}$

c. $\mathrm{E}(X) = 0.5$

d. $Var(1 - 2X) = 0.2$

e. $P(0.2 < X < 0.8) = \frac{99}{125} = 0.792.$

f. $M = 0.5$

Moment Generating Functions

25. $M_X(0) \neq 1$.

26. $M_X(t) = \frac{2\,e^t}{3-e^t}$. , mean = 1.5 and variance = 0.75.

27. $M_Z(t) = e^{0.5t^2}$ mean of X = 5 variance of X = 4 .

28. The p.d.f. of the r.v. X is given by

$$f(x) = ke^{-|x|}\,, -\infty < x < \infty$$

a. $k = 0.5$.

b. $P(-1 < X < 2) = 1 - 0.5e^{-2} - 0.5e^{-1}$.

$$F(x) = \begin{cases} 0.5\,e^{x} & x < 0 \\ 1 - 0.5\,e^{-x} & x > 0 \end{cases}$$

d. Mean = 0 and the standard deviation = $\sqrt{2}$.

29. The p.d.f of r.v Z is given by

$$f(z) = ke^{-2|z|} \qquad -\infty < z < \infty$$

a. k = 1.

b. $P(-2 < Z < 4) = 1 - 0.5e^{-8} - 0.5e^{-2}$.

$$F(z) = \begin{cases} 0.5\,e^{2z} & z < 0 \\ 1 - 0.5\,e^{-2z} & z > 0 \end{cases}$$

c. $M_Z(t) = \frac{4}{4-t^2}$, $-2 < t < 2$

d. Mean = 0 and standard deviation = $1/\sqrt{2}$

30. Hint $M_X(0) = 1$. $R'_X(t) = \frac{M'_X(t)}{M_X(t)}$.

31. $M_X(t) = \frac{1}{1-t}$, $t < 1$, mean =1 , variance = 1.

CHAPTER 4: SPECAIL PROBABILITY DISTRIBUTIONS

Discrete Distributions

1. **a.** 0.402 **b.** 0.402 **c.** 0.196
2. $n = 25$ and $p = 0.4$.
3. $n = 4$ and $p = 0.8$.
4. n = 5 and p = 0.4
5. $P(X \leq -17) = 0$
6. $X \sim \text{Bin}(7, 0.6)$ $P(X \leq 5) = 0.84147.$
7. $X \sim \text{Bin}(7, 0.2)$ $P(X \leq 5) = 0.99963.$
8. $X \sim \text{Bin}(5, 0.1)$ $P(X \geq 2) = 0.08146.$
9. $X \sim \text{Bin}(7, 0.6)$ **a.** $P(X = 4) = 0.290304$ **b.** $P(X \geq 5) = 0.419904$

10. $X \sim \text{Bin}(20, 0.85)$
    a. $P(X = 4) =$0.242829 **b.** $P(X \geq 15) =$ 0.932692

11. $X \sim \text{Bin}(5, 0.3)$
    a. $P(X = 3) =$ 0.1323 **b.** $P(X \geq 1) = 1 - P(X = 0) =$ 0.83193

12. $X \sim \text{Bin}(20, 0.1)$ P(X≤3)=0.867067

13. $X \sim \text{Bin}(5, 0.8)$ P(X≤4)=1-P(X=5) = 0.67232

14. $X \sim \text{Bin}(5, 0.6)$ P(X≥2)=1-P(X<2) = 0.91296

15.
    a. $X \sim \text{Bin}(20, 0.2)$ P(X≥5) = 0.37035 ≤ 0.5, $x = 5$

    b. $Y \sim \text{Bin}(20, 0.8)$ P(Y≥16) = 0.62965 ≥ 0.5 y = 16.

    Another solution: $P(Y \geq y) = P(X \leq 20 - y)$ since P(X≥5) = 0.37035 ≤ 0.5
    Therefore 1 − P(X≥5) = 1 - 0.37035 = 0.62965= P(X<5) = P(X≤4) ≥ 0.5
    Thus 20-y = 4 , y = 16 .

16.
$$\frac{(n-x)p}{(x+1)q}.\, b(x;n,p) = \frac{(n-x)p}{(x+1)q}\ {}^{n}C_x\, p^x q^{n-x} = \frac{(n-x)}{(x+1)}\ \frac{n!}{(n-x)!\quad x!}\ p^{x+1} q^{n-x-1} =$$
$$\frac{n!}{(n-x-1)!\quad (x+1)!}\ p^{x+1} q^{n-x-1} = {}^{n}C_{x+1}\, p^{x+1} q^{n-(x+1)} = b(x+1;n,p).$$

17. $X \sim HYP(n, N, M)$ $, n = 5$ $N = 40$ $M = 8$

$$P(X \leq 2) = \sum_{x=0}^{2} \frac{{}^{8}C_x \; {}^{32}C_{5-x}}{{}^{40}C_5}$$

18. $X \sim HYP(n, N, M)$ $, n = 10$ $N = 100$ $M = 20$

$$P(X \leq 3) = \sum_{x=0}^{3} \frac{{}^{20}C_x \; {}^{80}C_{10-x}}{{}^{100}C_{10}}$$

19.

$$h(x+1; n, M, N) = \frac{{}^{M}C_{x+1} \; {}^{N}C_{n-x-1}}{{}^{M+N}C_n} = \frac{1}{{}^{M+N}C_n} \frac{M!}{(x+1)! \; (M-x-1)!} \; \frac{N!}{(n-x-1)! \; (N-n+x+1)!} =$$
$$\frac{1}{{}^{M+N}C_n} \frac{(M-x-1) \quad M!}{(x+1) \; x! \quad (M-x)!} \; \frac{(n-x) \qquad N!}{(N-n+x+1) \quad (n-x)! \; (N-n+x)!} =$$
$$\frac{1}{{}^{M+N}C_n} \frac{(M-x-1) \quad {}^{M}C_x}{(x+1)} \; \frac{(n-x) \qquad {}^{N}C_{n-x}}{(N-n+x+1)} = \frac{(M-x-1) \; (n-x)}{(x+1) \; (N-n+x+1)} \; \frac{{}^{M}C_x \; {}^{N}C_{n-x}}{{}^{M+N}C_n} =$$
$$\frac{(n-x)(M-x)}{(x+1)(N-M-n+x+1)} \, . h(x; n, M, N)$$

20. X is a r.v denotes the number of times needed to turn on and off the light switch until it fails.

$$X \sim Geo(p) \quad , p = 0.001$$
$$P(X > 800) = (0.999)^{800}$$

21.

a. $X \sim Geo(p)$ $, p = 0.75$ $P(X = 5) = 0.75\,(0.25)^4$

b. $X \sim NBin(k, p)$ $, k = 3,\ p = 0.75$
$P(X = 8) = {}^{7}C_2\,(0.75)^3\,(0.25)^5$

22. $P(X > 1) = 0.25$.

23. $p = 0.4,$ $P(X > 2) = 1 - P(X \leq 2) = 0.36$.

24.

$$P(X = m + n / X > n) = \frac{P(X=m+n)}{P(X>n)} = \frac{p\, q^{m+n-1}}{q^n} = p\, q^{m-1} = P(X = m)$$

25.

$$\sum_{x=1}^{\infty} p(1-p)^{x-1} = 1 \quad \rightarrow \quad \sum_{x=1}^{\infty} (1-p)^{x-1} = \frac{1}{p} \rightarrow \sum_{x=0}^{\infty} (1-p)^{x} = \frac{1}{p}$$

Differentiating w.r.t p

$$\sum_{x=1}^{\infty} x\,(1-p)^{x-1} = \frac{1}{p^2}$$

Multiply both sides by p

$$\sum_{x=1}^{\infty} x\, p\, (1-p)^{x-1} = \frac{p}{p^2} = \frac{1}{p}$$

Since $\mu = \sum_{x=1}^{\infty} x\, p\, (1-p)^{x-1}$ therefore $\mu = \frac{1}{p}$

26. $P(Y > 0) = 0.75$

27. $\mathrm{P(X} > 2) = 1 - 2e^{-1}$.

28. $\mu_x = \sigma_x^2 = \ln(2)$.

29. $P(X > 1) = 1 - 1.5\, e^{-0.5}$.

30. Mean= 6.

31. E(X) = 2.

32.

$$X \sim Bin(n, p) \quad n = 200\; p = 0.02$$
$$X \sim Pois(\lambda) \quad \lambda = np = 4$$
$$P(X \geq 4) = 1 - \sum_{x=0}^{4} \frac{e^{-4}\, 4^x}{x\,!}$$

33. $P(X = 5) = \frac{e^{-2}\;2^5}{5!} = 0.0360894$

34. [Ans = 0.0613]

35. [Ans = 0.0916]

36. $X \sim Pois(\lambda) \quad \lambda = 40 \qquad P(X \leq 10) = \sum_{x=0}^{10} \frac{e^{-40}\; 40^x}{x!}$

37.

a. $e^{-1.8}$ **b.** $1.8\, e^{-1.8}$

38. $P(X = 6) = \frac{e^{-4}\;4^6}{6!} = 0.1041956$

$P(X \geq 10) = 1 - \sum_{x=0}^{9} \frac{e^{-10}\; 10^x}{x!} = 0.54207$

39. [Ans = 0.7619 ; 0.3829]

40. [Ans. = 0.7586]

Continuous Distributions

41. a)1/30 b) 2/3

42. $k = 10.$

43. c = -0.3

44.

- **a.** k = 1/6
- **b.** $P(X > 0) = 4/6$
- **c.** $P(-1 < X < 3) = 4/6$
- **d.** $P(X < 2)$= 4/6

45.

$$P(X > \alpha + \beta \ / \ X > \ \alpha) = \frac{P\ (X > \alpha + \beta \ \text{ and } \ X > \ \alpha)}{P(X > \ \alpha)}$$

$$= \frac{P\ (X > \alpha + \beta)}{P(X > \ \alpha)} = \frac{e^{-(\alpha+\beta)}}{e^{-\alpha}} = \frac{e^{-\alpha}\ \ e^{-\beta}}{e^{-\alpha}} = \ \ e^{-\beta} \ = P(X > \ \beta)$$

46. $P(X > \ 1 \ / \ X > 2)$ = 1

47. P(X< 1 / X<2) = $\frac{1-e^{-0.5}}{1-e^{-1}}$.

48. $X \sim EXP(\theta) \ \ ; \theta = 5$

- **a.** $E(X) = 5$
- **b.** Variance =25
- **c.** S.D = 5
- **d.** $E(X + 5)^2 = 125.$

49. $X \sim EXP(\theta) \ \ ; \theta = 900$

- **a.** mean = 900.
- **b.** Variance = 810000 and S.D =900.

50. [Ans. = 0.6065]

51. $P(Z > A) = 0.3.$

52. $\mu = 32.1$

53. $P(Z > c) = 0.2$.

54. σ_x = 10

55. $\mu = 39$

56. P (360 < X < 469) =0.7043 and P(X = 400) = zero,

57. P($|X|$< 1) = 0.2586

58. Mean = 98.9875

59. Number of students you expect to have heights between 66 and 72 inches ≅ 266

60. [Ans. = 0.511].

61.

- **a.** 26.595%
- **b.** v =108.24 km/h.

62.

- **a.** 0.0228
- **b.** mean =1.02325

63. [Ans. = 6.68%]

64. [Ans. = 0.392].

65.

- **a.** 6.68%
- **b.** 24.17%

66. probability that the satellite fails= 1- 0.9876 = 0.0124.