

Chapter 38

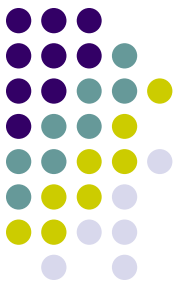
Diffraction Patterns and Polarization





Diffraction

- Light of wavelength comparable to or larger than the width of a slit spreads out in all forward directions upon passing through the slit
- This phenomena is called *diffraction*
 - This indicates that light spreads beyond the narrow path defined by the slit into regions that would be in shadow if light traveled in straight lines



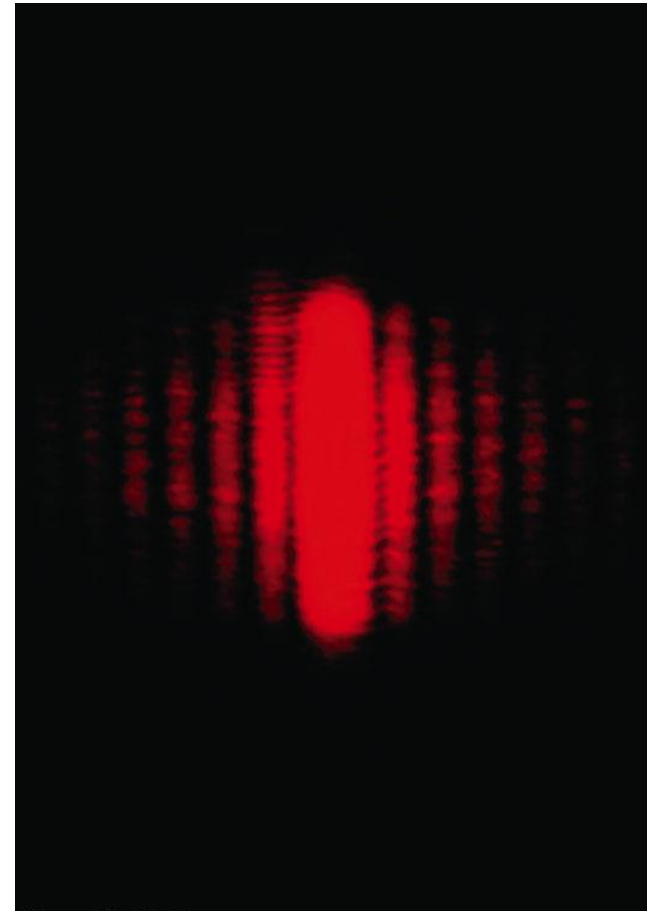
Diffraction Pattern

- A single slit placed between a distant light source and a screen produces a **diffraction pattern**
 - It will have a broad, intense central band
 - Called the **central maximum**
 - The central band will be flanked by a series of narrower, less intense secondary bands
 - Called **side maxima** or **secondary maxima**
 - The central band will also be flanked by a series of dark bands
 - Called **minima**



Diffraction Pattern, Single Slit

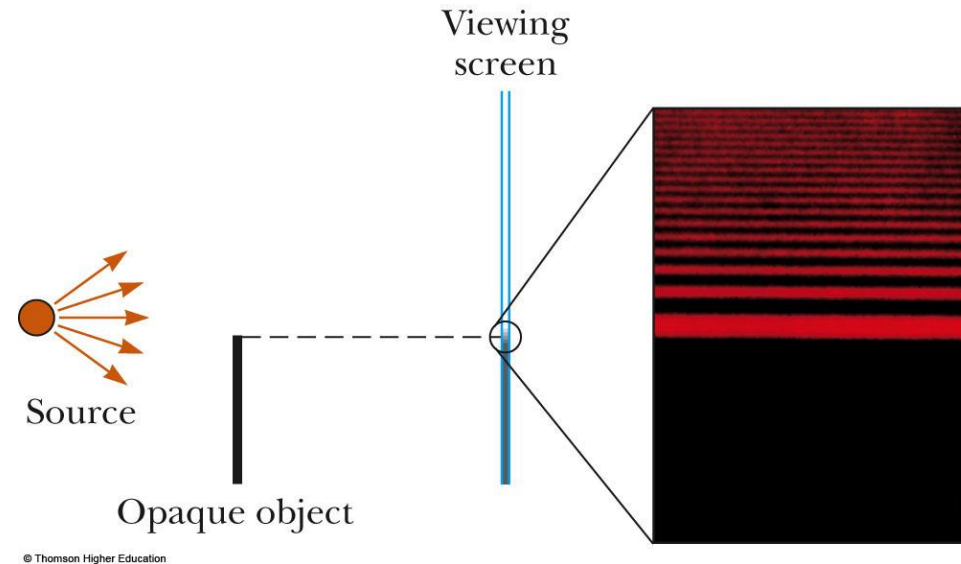
- The diffraction pattern consists of the central maximum and a series of secondary maxima and minima
- The pattern is similar to an interference pattern



Diffraction Pattern, Object Edge



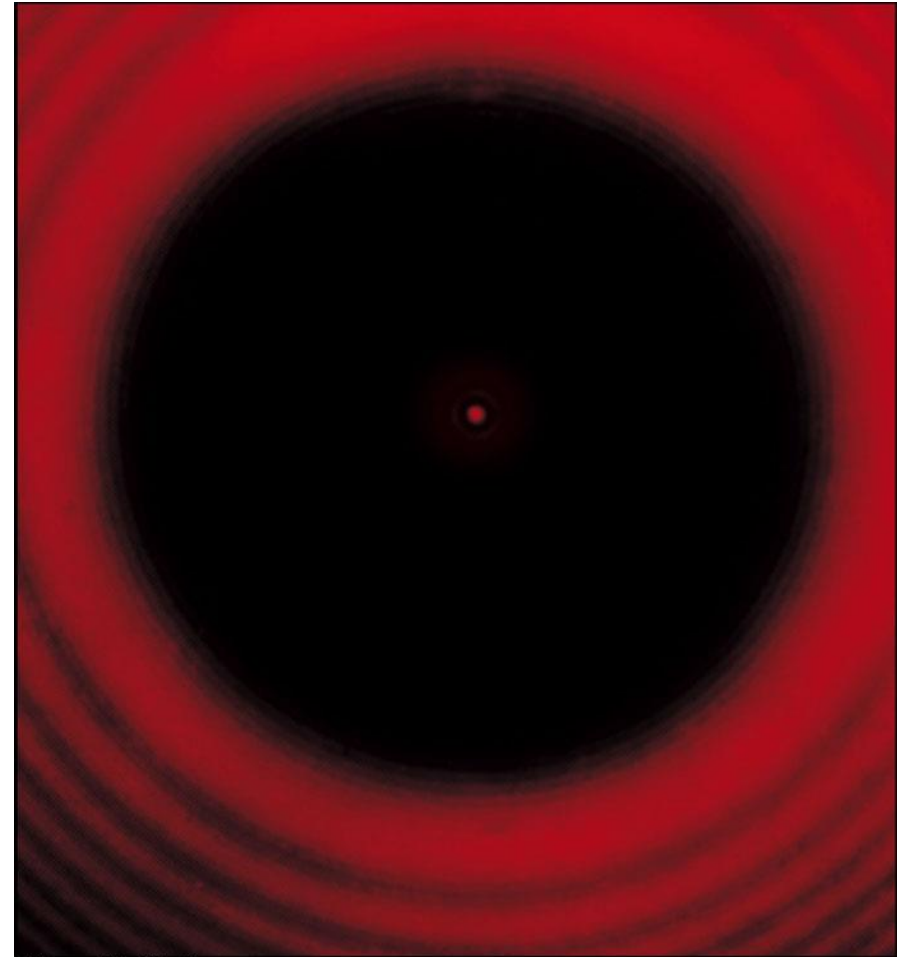
- This shows the upper half of the diffraction pattern formed by light from a single source passing by the edge of an opaque object
- The diffraction pattern is vertical with the central maximum at the bottom

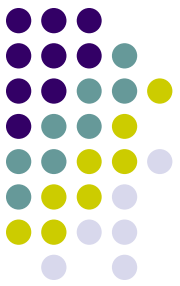




Confirming Wave Nature

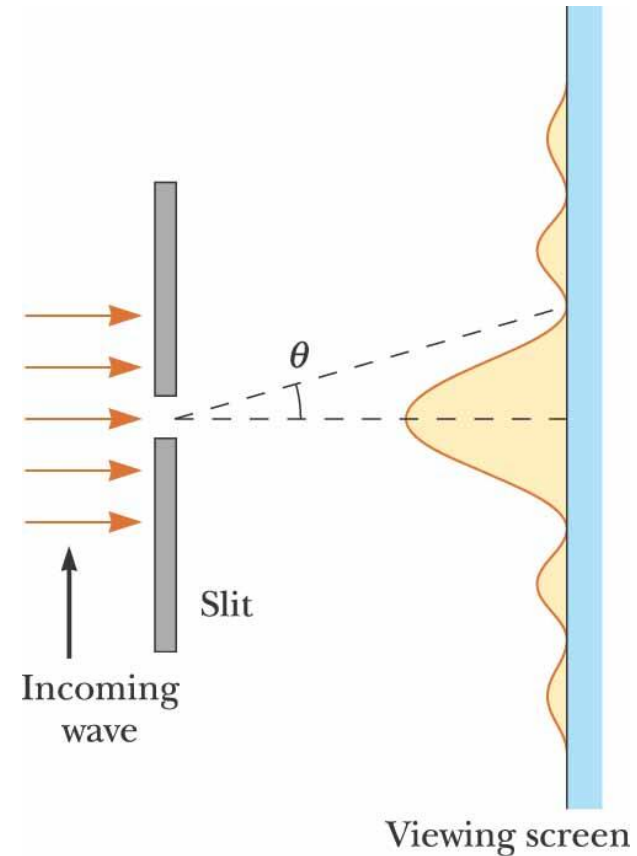
- Geometric optics would predict a dark spot in the center
- Wave theory predicts the presence of the center spot
- There is a bright spot at the center
 - Confirms wave theory
- The circular fringes extend outward from the shadow's edge





Fraunhofer Diffraction Pattern

- A **Fraunhofer diffraction pattern** occurs when the rays leave the diffracting object in parallel directions
 - Screen very far from the slit
 - Could be accomplished by a converging lens



Fraunhofer Diffraction Pattern Photo



- A bright fringe is seen along the axis ($\theta = 0$)
- Alternating bright and dark fringes are seen on each side

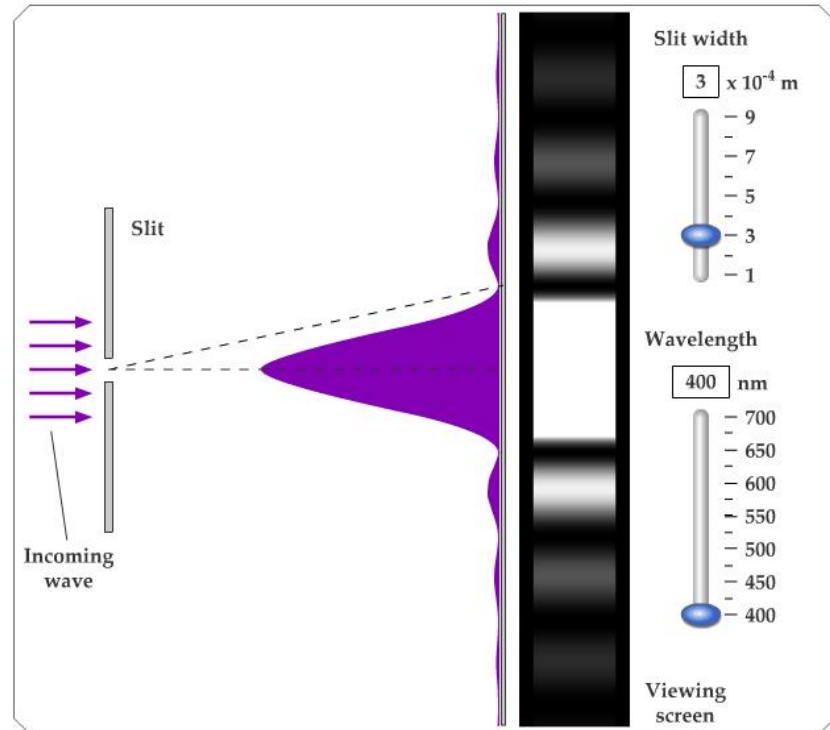


(b)

Active Figure 38.4



- Use the active figure to adjust the slit width and the wavelength of the light
- Observe the effect on the diffraction pattern



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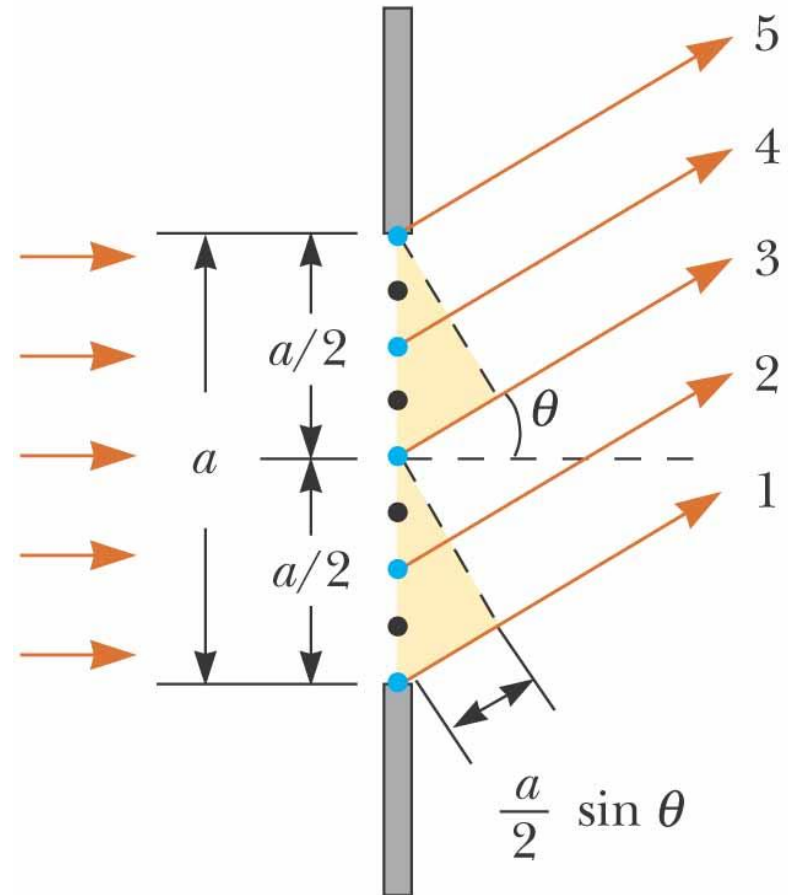
Diffraction vs. Diffraction Pattern



- *Diffraction* refers to the general behavior of waves spreading out as they pass through a slit
- A *diffraction pattern* is actually a misnomer that is deeply entrenched
 - The pattern seen on the screen is actually another *interference* pattern
 - The interference is between parts of the incident light illuminating different regions of the slit

Single-Slit Diffraction

- The finite width of slits is the basis for understanding Fraunhofer diffraction
- According to Huygens's principle, each portion of the slit acts as a source of light waves
- Therefore, light from one portion of the slit can interfere with light from another portion



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Single-Slit Diffraction, 2

- The resultant light intensity on a viewing screen depends on the direction θ
- The diffraction pattern is actually an interference pattern
 - The different sources of light are different portions of the single slit

Single-Slit Diffraction, Analysis

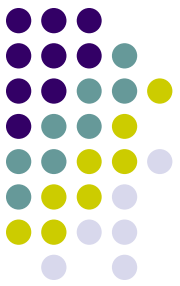


- All the waves are in phase as they leave the slit
- Wave 1 travels farther than wave 3 by an amount equal to the path difference
 - $(a/2) \sin \theta$
- If this path difference is exactly half of a wavelength, the two waves cancel each other and destructive interference results
- In general, destructive interference occurs for a single slit of width a when $\sin \theta_{\text{dark}} = m\lambda / a$
 - $m = \pm 1, \pm 2, \pm 3, \dots$

Single-Slit Diffraction, Intensity



- The general features of the intensity distribution are shown
- A broad central bright fringe is flanked by much weaker bright fringes alternating with dark fringes
- Each bright fringe peak lies approximately halfway between the dark fringes
- The central bright maximum is twice as wide as the secondary maxima



Intensity, equation

- The intensity can be expressed as

$$I = I_{\max} \left[\frac{\sin(\pi a \sin \theta / \lambda)}{\pi a \sin \theta / \lambda} \right]^2$$

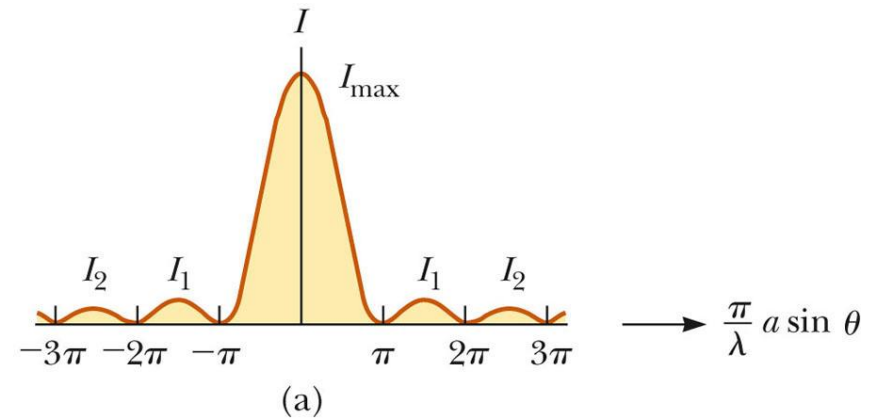
- Minima occur at

$$\frac{\pi a \sin \theta_{\text{dark}}}{\lambda} = m\pi \quad \text{or} \quad \sin \theta_{\text{dark}} = m \frac{\lambda}{a}$$

Intensity, final



- Most of the light intensity is concentrated in the central maximum
- The graph shows a plot of light intensity vs. $(\pi / \lambda) a \sin \theta$



(b)

Intensity of Two-Slit Diffraction Patterns



- When more than one slit is present, consideration must be made of
 - The diffraction patterns due to individual slits
 - The interference due to the wave coming from different slits
- The single-slit diffraction pattern will act as an “envelope” for a two-slit interference pattern

Intensity of Two-Slit Diffraction Patterns, Equation



- To determine the maximum intensity:

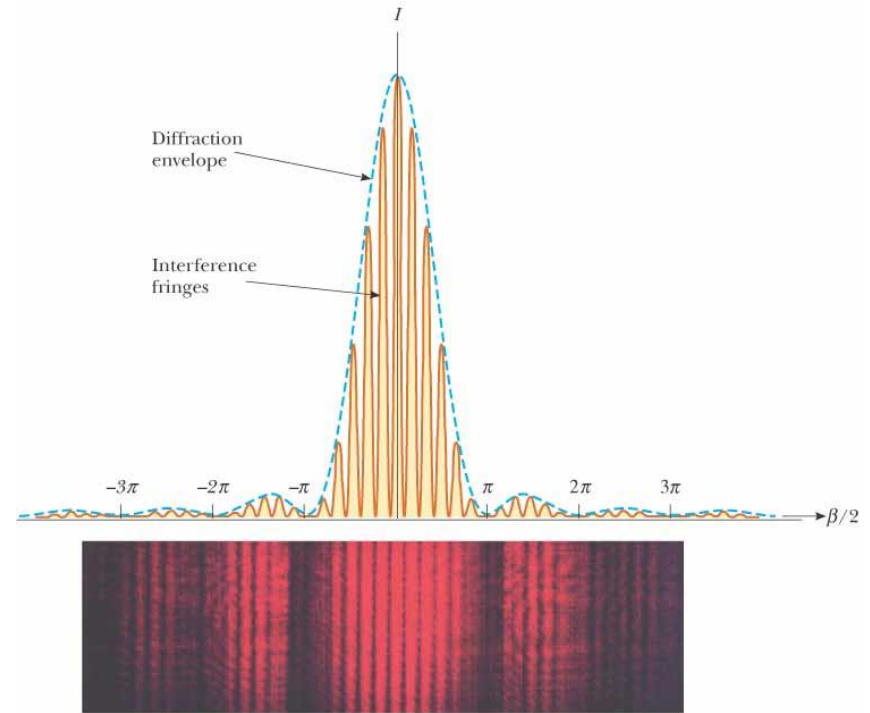
$$I = I_{\max} \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right) \left[\frac{\sin(\pi a \sin \theta / \lambda)}{\pi a \sin \theta / \lambda} \right]^2$$

- The factor in the square brackets represents the single-slit diffraction pattern
 - This acts as the envelope
- The two-slit interference term is the \cos^2 term

Intensity of Two-Slit Diffraction Patterns, Graph of Pattern



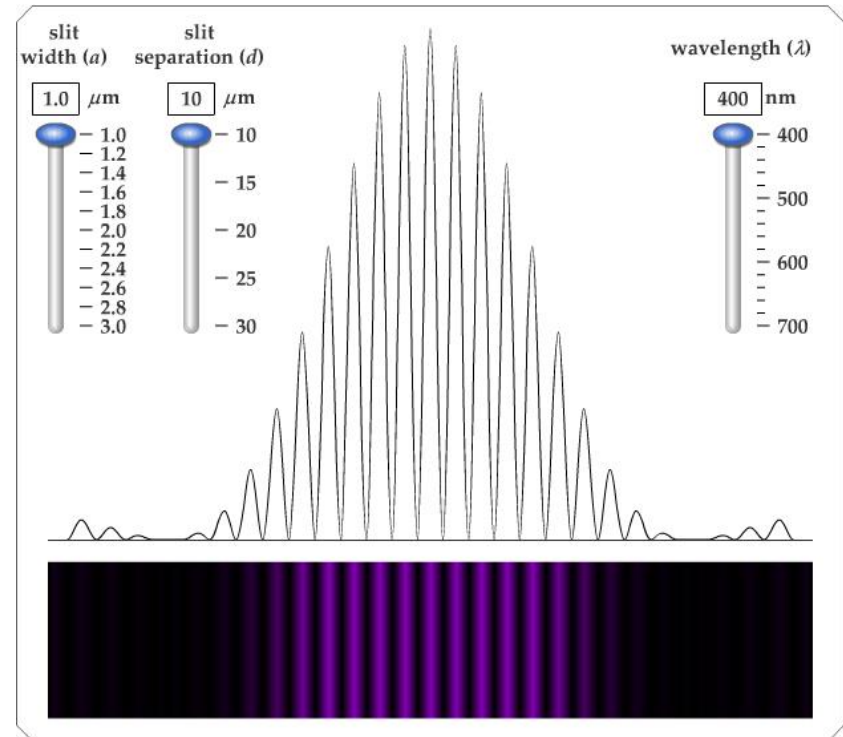
- The broken blue line is the diffraction pattern
- The brown curve shows the \cos^2 term
 - This term, by itself, would result in peaks with all the same heights
 - The uneven heights result from the diffraction term (square brackets in the equation)



Active Figure 38.7



- Use the active figure to adjust
 - The slit width
 - The slit separation
 - The wavelength of the light
- Observe the effect on the interference pattern



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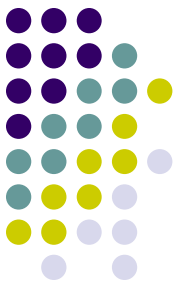
Two-Slit Diffraction Patterns, Maxima and Minima



- To find which interference maximum coincides with the first diffraction minimum

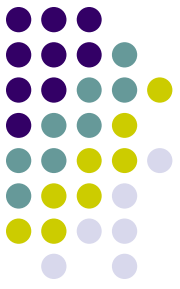
$$\frac{d \sin \theta}{a \sin \theta} = \frac{m\lambda}{\lambda} \rightarrow \frac{d}{a} = m$$

- The conditions for the first interference maximum
 - $d \sin \theta = m\lambda$
- The conditions for the first diffraction minimum
 - $a \sin \theta = \lambda$



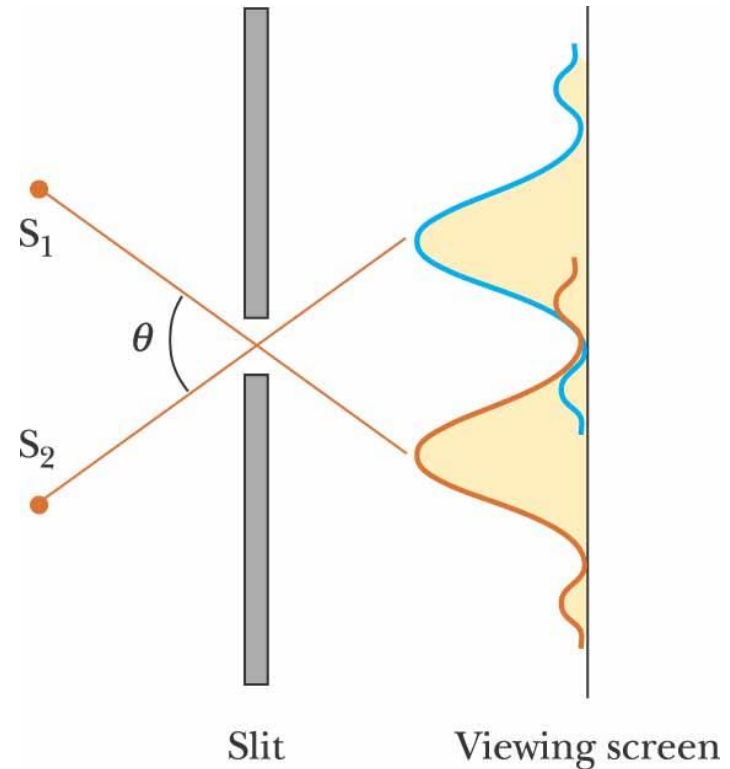
Resolution

- The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light
- If two sources are far enough apart to keep their central maxima from overlapping, their images can be distinguished
 - The images are said to be resolved
- If the two sources are close together, the two central maxima overlap and the images are not resolved

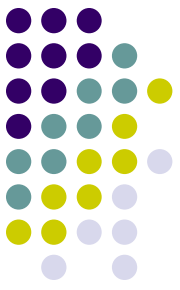


Resolved Images, Example

- The images are far enough apart to keep their central maxima from overlapping
- The angle subtended by the sources at the slit is large enough for the diffraction patterns to be distinguishable
- The images are resolved

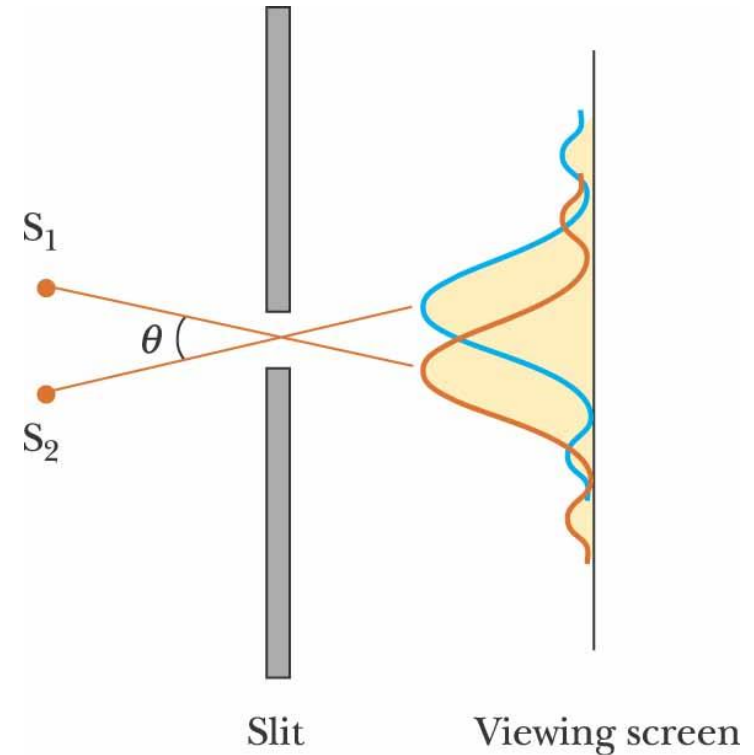


(a)



Images Not Resolved, Example

- The sources are so close together that their central maxima do overlap
- The angle subtended by the sources is so small that their diffraction patterns overlap
- The images are not resolved



(b)

Resolution, Rayleigh's Criterion



- When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved
- This limiting condition of resolution is called **Rayleigh's criterion**

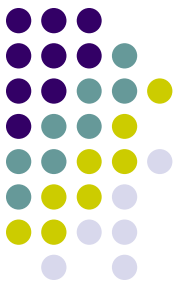
Resolution, Rayleigh's Criterion, Equation



- The angle of separation, θ_{\min} , is the angle subtended by the sources for which the images are just resolved
- Since $\lambda \ll a$ in most situations, $\sin \theta$ is very small and $\sin \theta \approx \theta$
- Therefore, the limiting angle (in rad) of resolution for a slit of width a is

$$\theta_{\min} = \lambda / a$$

- To be resolved, the angle subtended by the two sources must be greater than θ_{\min}



Circular Apertures

- The diffraction pattern of a circular aperture consists of a central bright disk surrounded by progressively fainter bright and dark rings
- The limiting angle of resolution of the circular aperture is

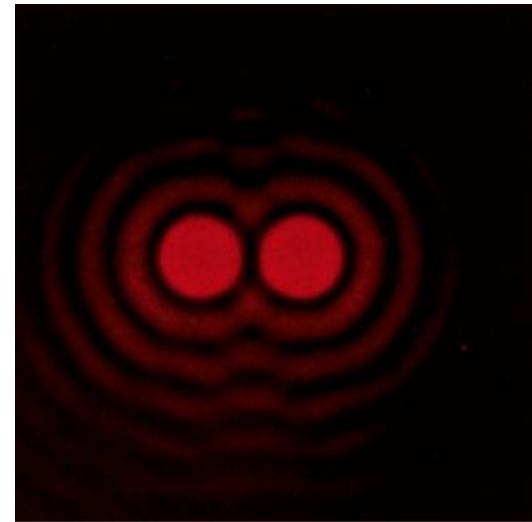
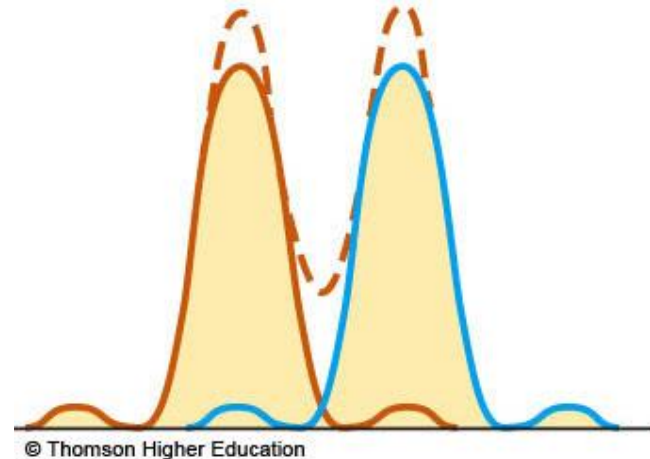
$$\theta_{\min} = 1.22 \frac{\lambda}{D}$$

- D is the diameter of the aperture

Circular Apertures, Well Resolved



- The sources are far apart
- The images are well resolved
- The solid curves are the individual diffraction patterns
- The dashed lines are the resultant pattern

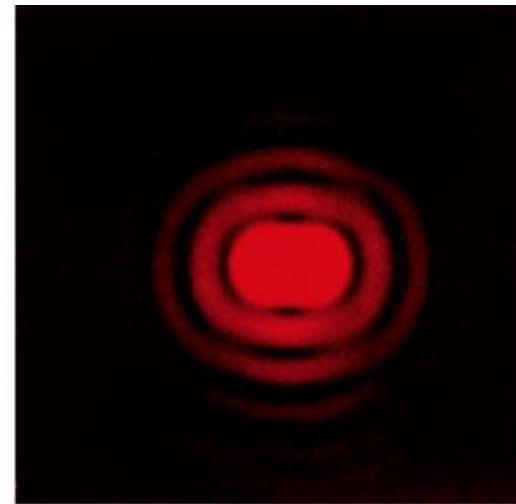
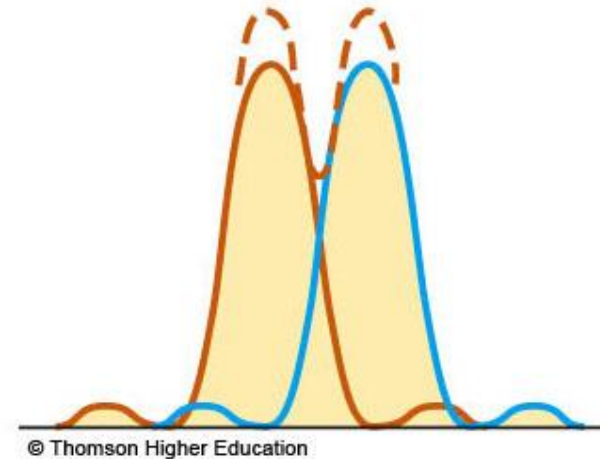


(a)

Circular Apertures, Just Resolved



- The sources are separated by an angle that satisfies Rayleigh's criterion
- The images are just resolved
- The solid curves are the individual diffraction patterns
- The dashed lines are the resultant pattern

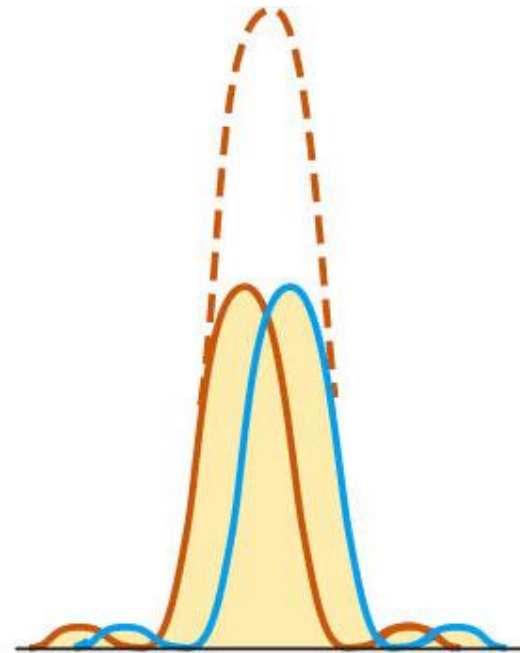


(b)

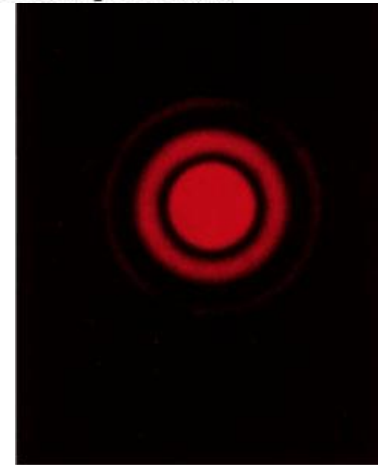
Circular Apertures, Not Resolved



- The sources are close together
- The images are unresolved
- The solid curves are the individual diffraction patterns
- The dashed lines are the resultant pattern

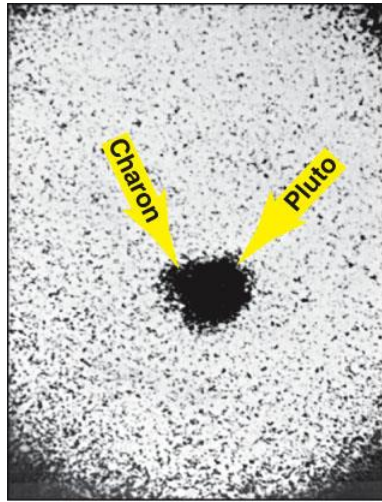


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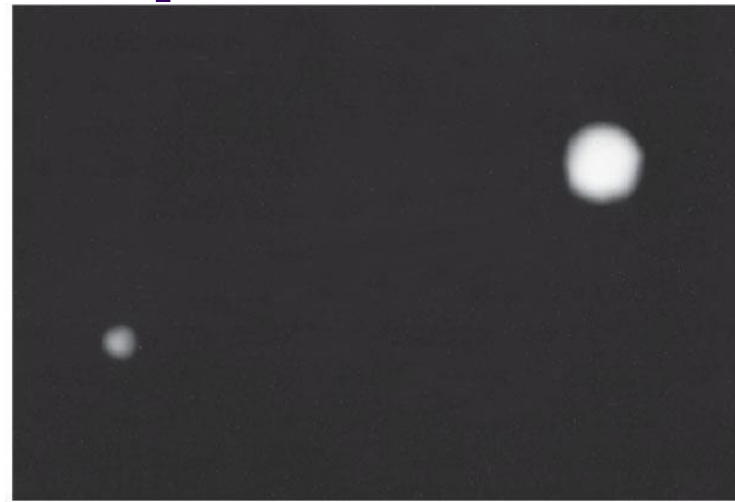
(c)

Resolution, Example



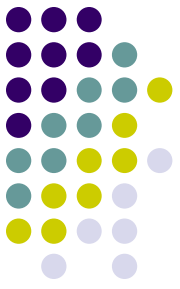
(a)

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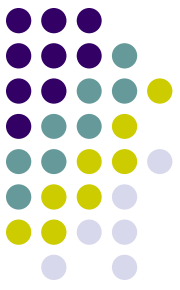
(b)

- Pluto and its moon, Charon
- Left: Earth-based telescope is blurred
- Right: Hubble Space Telescope clearly resolves the two objects



Diffraction Grating

- The diffracting grating consists of a large number of equally spaced parallel slits
 - A typical grating contains several thousand lines per centimeter
- The intensity of the pattern on the screen is the result of the combined effects of interference and diffraction
 - Each slit produces diffraction, and the diffracted beams interfere with one another to form the final pattern



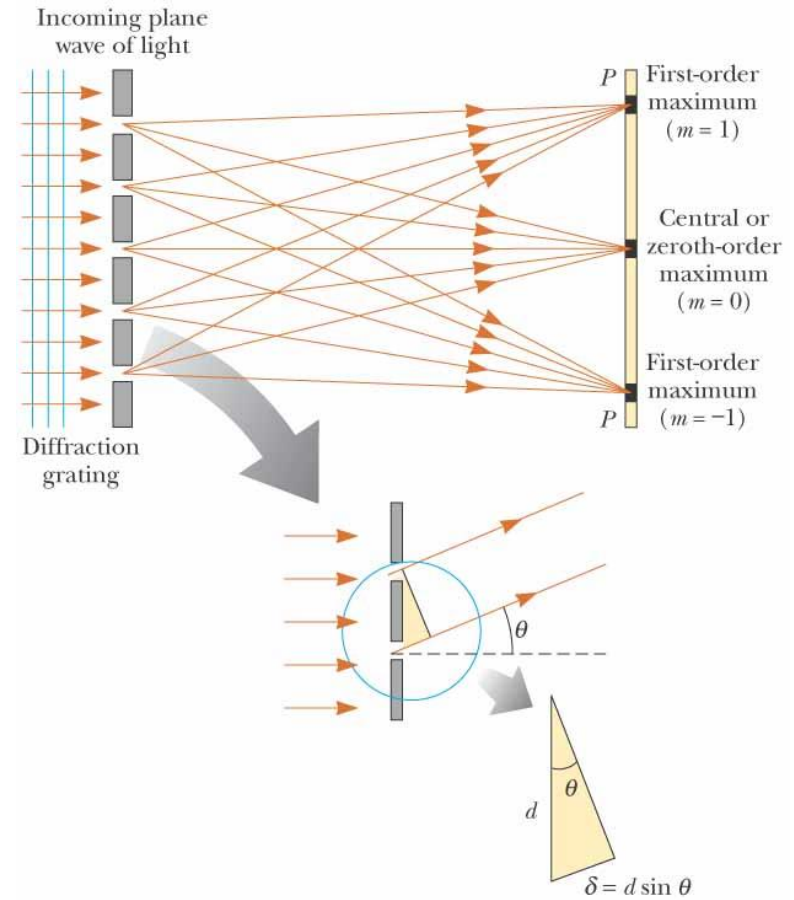
Diffraction Grating, Types

- A *transmission* grating can be made by cutting parallel grooves on a glass plate
 - The spaces between the grooves are transparent to the light and so act as separate slits
- A *reflection* grating can be made by cutting parallel grooves on the surface of a reflective material
 - The spaces between the grooves act as parallel sources of reflected light, like the slits in a transmission grating



Diffraction Grating, cont.

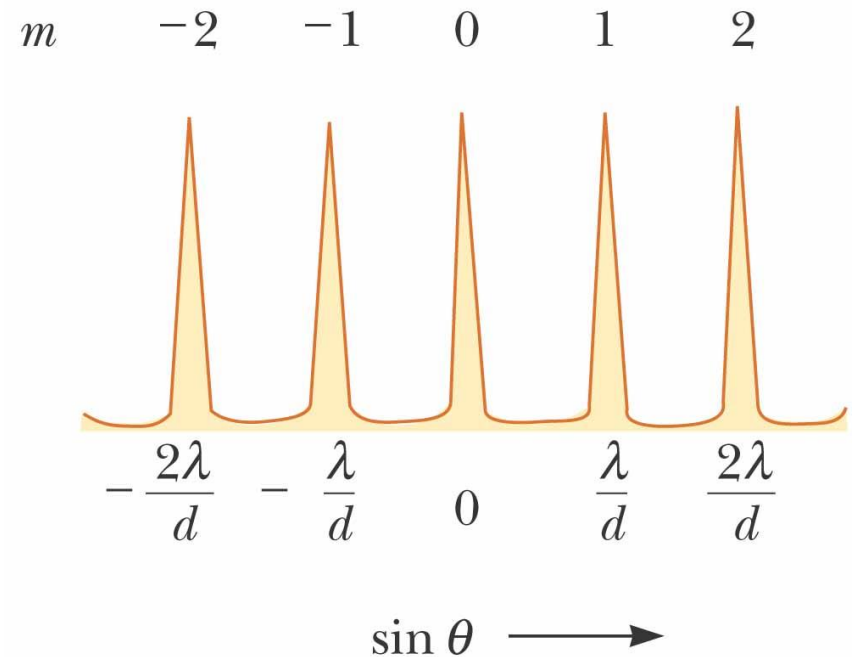
- The condition for *maxima* is
 - $d \sin \theta_{\text{bright}} = m\lambda$
 - $m = 0, \pm 1, \pm 2, \dots$
- The integer m is the *order number* of the diffraction pattern
- If the incident radiation contains several wavelengths, each wavelength deviates through a specific angle



Diffraction Grating, Intensity



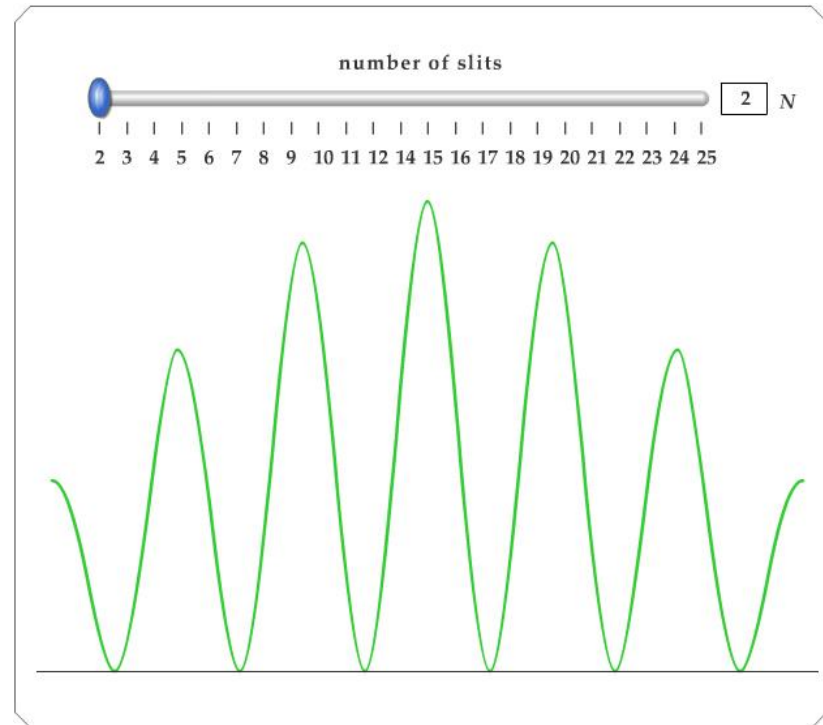
- All the wavelengths are seen at $m = 0$
 - This is called the zeroth-order maximum
- The first-order maximum corresponds to $m = 1$
- Note the sharpness of the principle maxima and the broad range of the dark areas



Active Figure 38.13



- Use the active figure to choose the number of slits
- See the effect on the interference pattern



**PLAY
ACTIVE FIGURE**

Diffraction Grating, Intensity, cont.

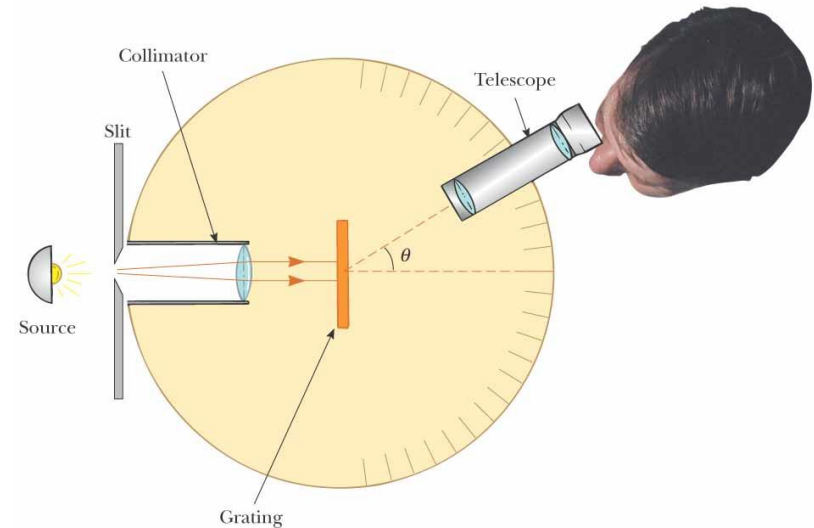


- Characteristics of the intensity pattern
 - The sharp peaks are in contrast to the broad, bright fringes characteristic of the two-slit interference pattern
 - Because the principle maxima are so sharp, they are much brighter than two-slit interference patterns

Diffraction Grating Spectrometer



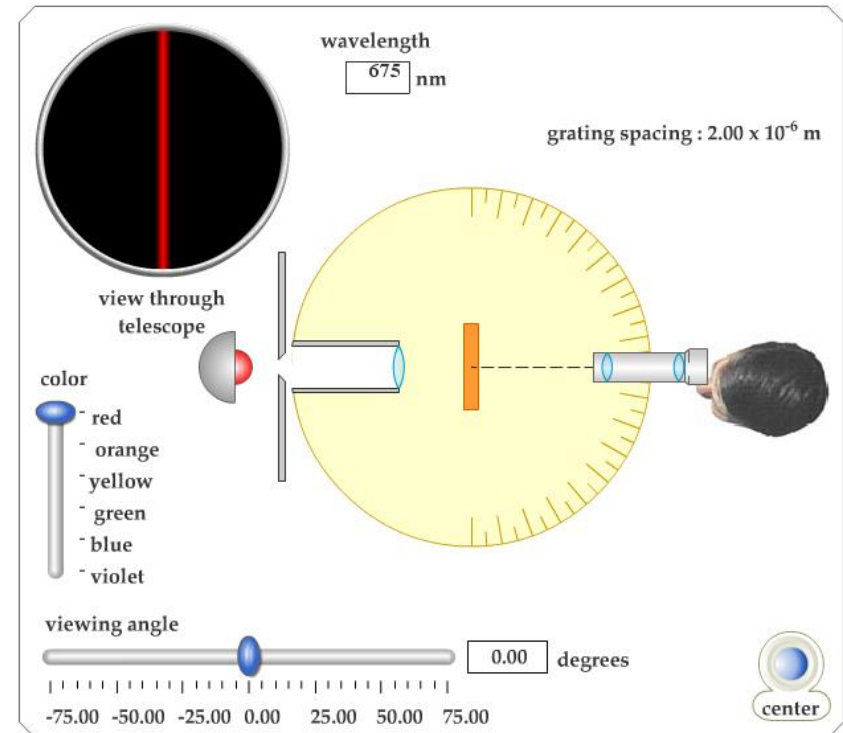
- The collimated beam is incident on the grating
- The diffracted light leaves the gratings and the telescope is used to view the image
- The wavelength can be determined by measuring the precise angles at which the images of the slit appear for the various orders



Active Figure 38.15



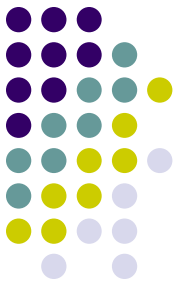
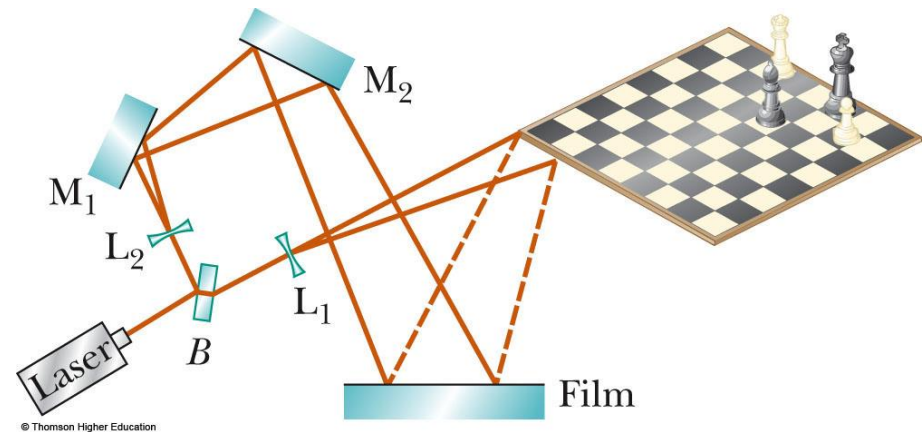
- Use the active figure to operate the spectrometer
- Observe constructive interference for various wavelengths



**PLAY
ACTIVE FIGURE**

Holography

- *Holography* is the production of three-dimensional images of objects
- Light from a laser is split into two parts by a half-silvered mirror at B
- One part of the light reflects off the object and strikes the film

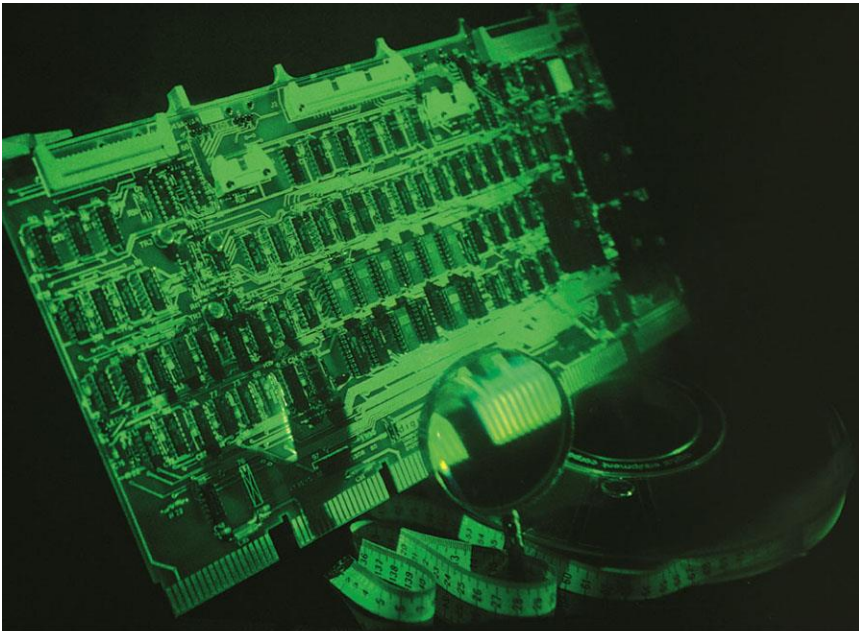


Holography, cont

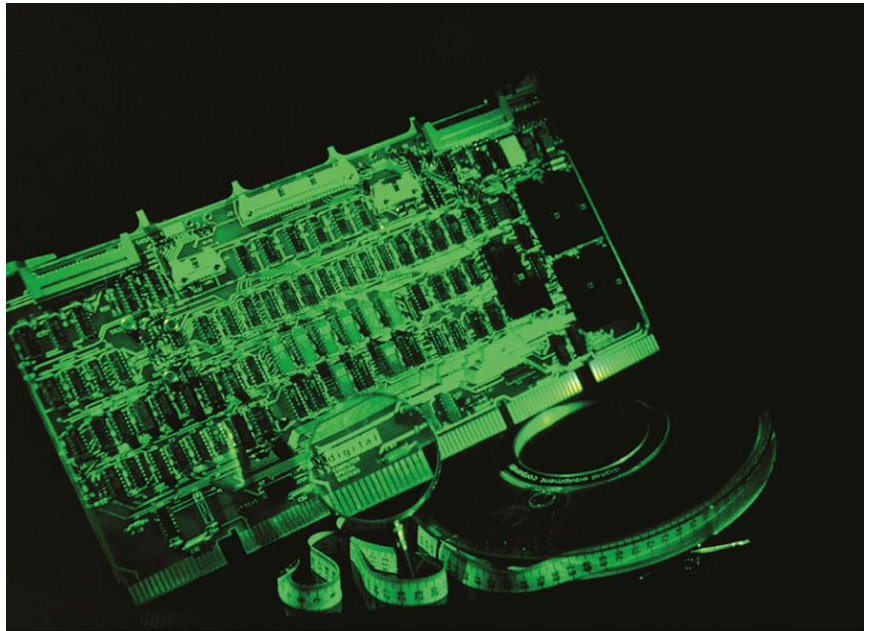


- The other half of the beam is diverged by lens L_2
- It then reflects to mirrors M_1 and M_2 and then strikes the film
- The two beams overlap to form a complex interference pattern on the film
- The holograph records the intensity of the light reflected by the object as well as the phase difference between the reference beam and the beam scattered from the object

Holography, Example

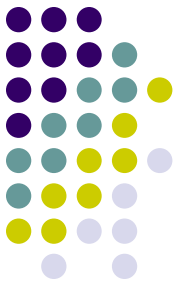


(a)



(b)

Diffraction of X-Rays by Crystals

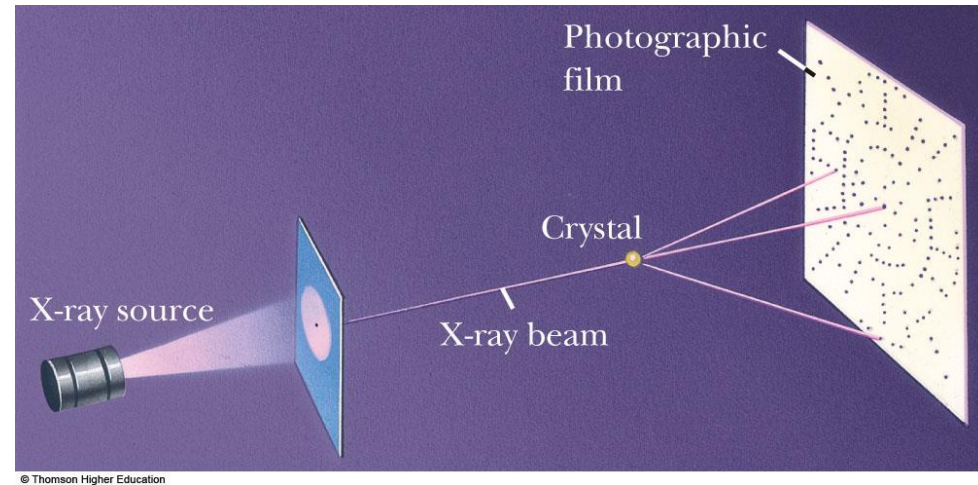


- X-rays are electromagnetic waves of very short wavelength
- Max von Laue suggested that the regular array of atoms in a crystal could act as a three-dimensional diffraction grating for x-rays

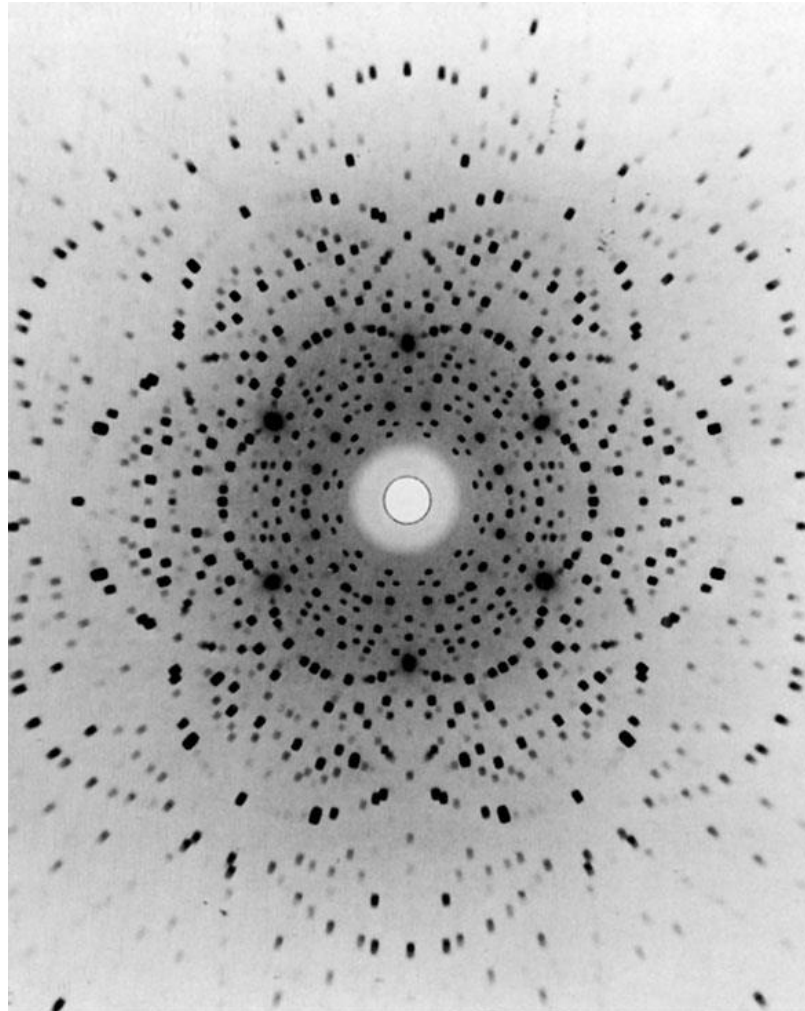
Diffraction of X-Rays by Crystals, Set-Up



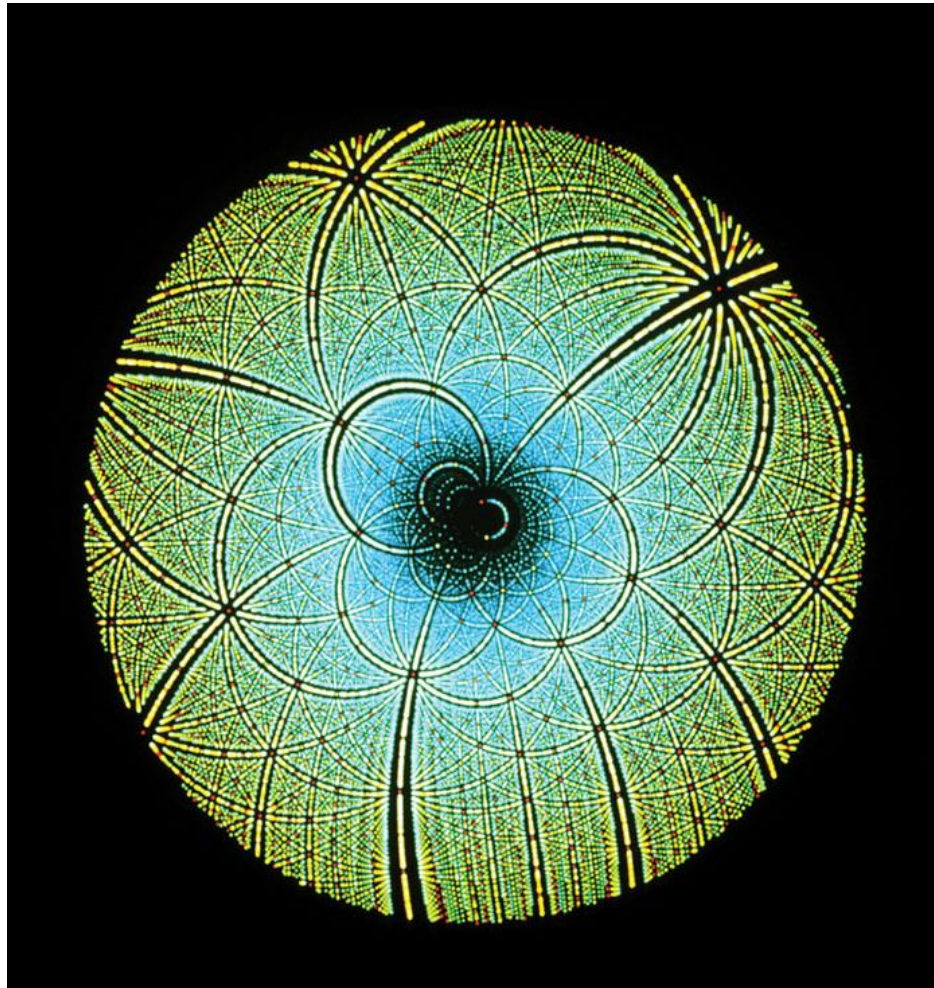
- A collimated beam of monochromatic x-rays is incident on a crystal
- The diffracted beams are very intense in certain directions
 - This corresponds to constructive interference from waves reflected from layers of atoms in the crystal
- The diffracted beams form an array of spots known as a *Laue pattern*



Laue Pattern for Beryl



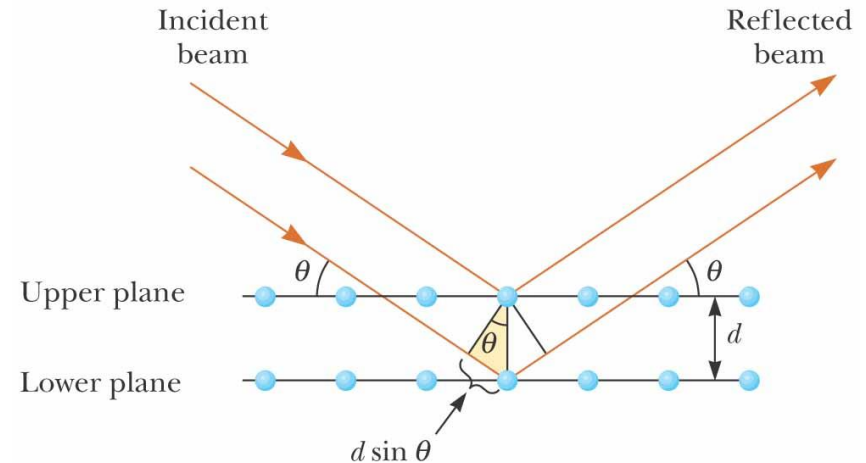
Laue Pattern for Rubisco



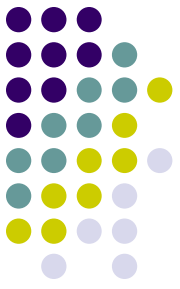
X-Ray Diffraction, Equations



- This is a two-dimensional description of the reflection of the x-ray beams
- The condition for *constructive interference* is
$$2d \sin \theta = m\lambda$$
where $m = 1, 2, 3$
- This condition is known as **Bragg's law**
- This can also be used to calculate the spacing between atomic planes

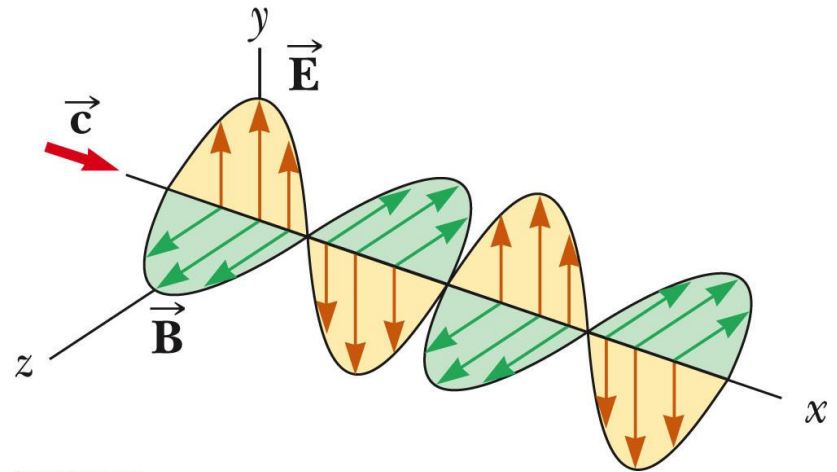


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Polarization of Light Waves

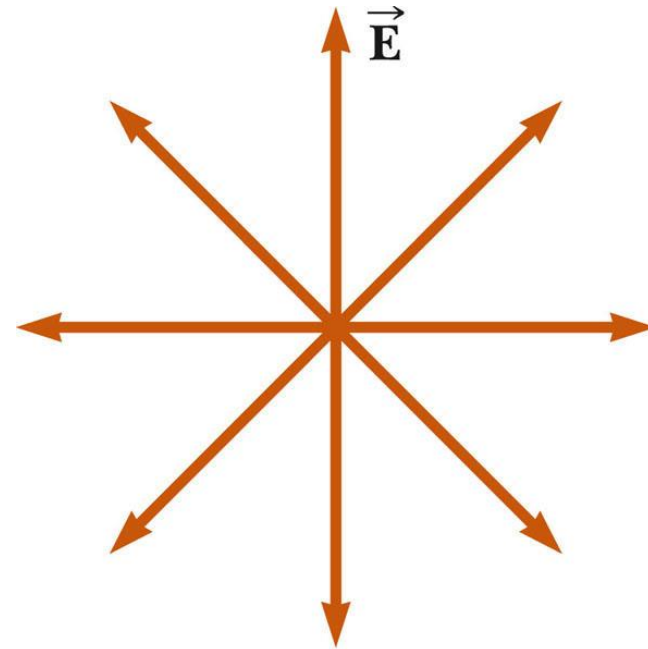
- The *direction of polarization* of each individual wave is defined to be the direction in which the electric field is vibrating
- In this example, the direction of polarization is along the y -axis





Unpolarized Light, Example

- All directions of vibration from a wave source are possible
- The resultant em wave is a superposition of waves vibrating in many different directions
- This is an unpolarized wave
- The arrows show a few possible directions of the waves in the beam



(a)



Polarization of Light, cont.

- A wave is said to be *linearly polarized* if the resultant electric field vibrates in the same direction *at all times* at a particular point
- The plane formed by E and the direction of propagation is called the *plane of polarization* of the wave



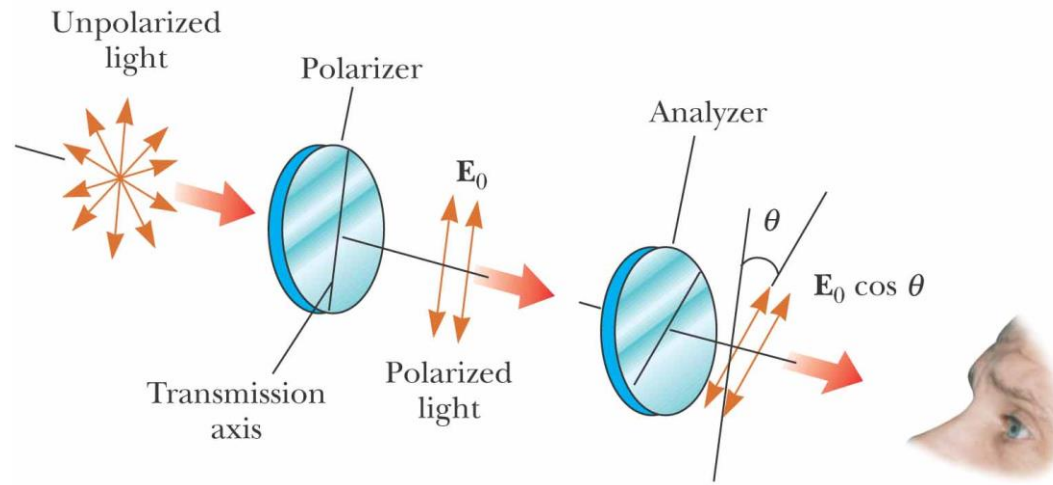
(b)



Methods of Polarization

- It is possible to obtain a linearly polarized beam from an unpolarized beam by removing all waves from the beam except those whose electric field vectors oscillate in a single plane
- Processes for accomplishing this include
 - Selective absorption
 - Reflection
 - Double refraction
 - Scattering

Polarization by Selective Absorption



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- The most common technique for polarizing light
- Uses a material that transmits waves whose electric field vectors lie in the plane parallel to a certain direction and absorbs waves whose electric field vectors are in all other directions

Selective Absorption, cont.

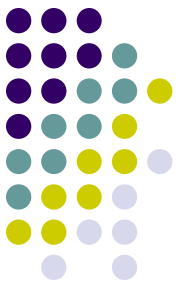


- E. H. Land discovered a material that polarizes light through selective absorption
 - He called the material *Polaroid*
 - The molecules readily absorb light whose electric field vector is parallel to their lengths and allow light through whose electric field vector is perpendicular to their lengths



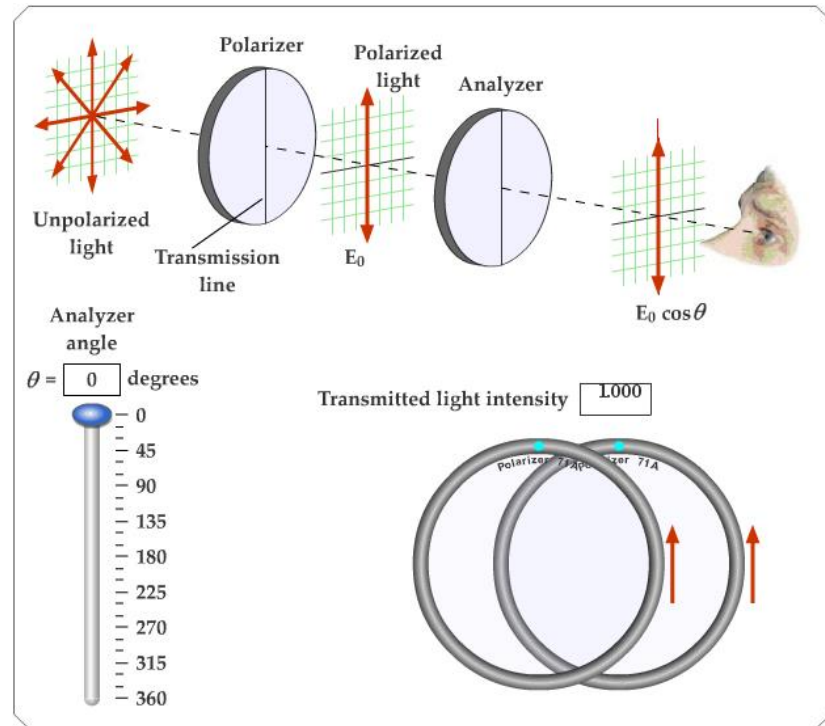
Selective Absorption, final

- It is common to refer to the direction perpendicular to the molecular chains as the *transmission axis*
- In an *ideal* polarizer,
 - All light with the electric field parallel to the transmission axis is transmitted
 - All light with the electric field perpendicular to the transmission axis is absorbed



Active Figure 38.26

- Use the active figure to rotate the analyzer
- Observe the effect on the transmitted light



**PLAY
ACTIVE FIGURE**

Intensity of a Polarized Beam



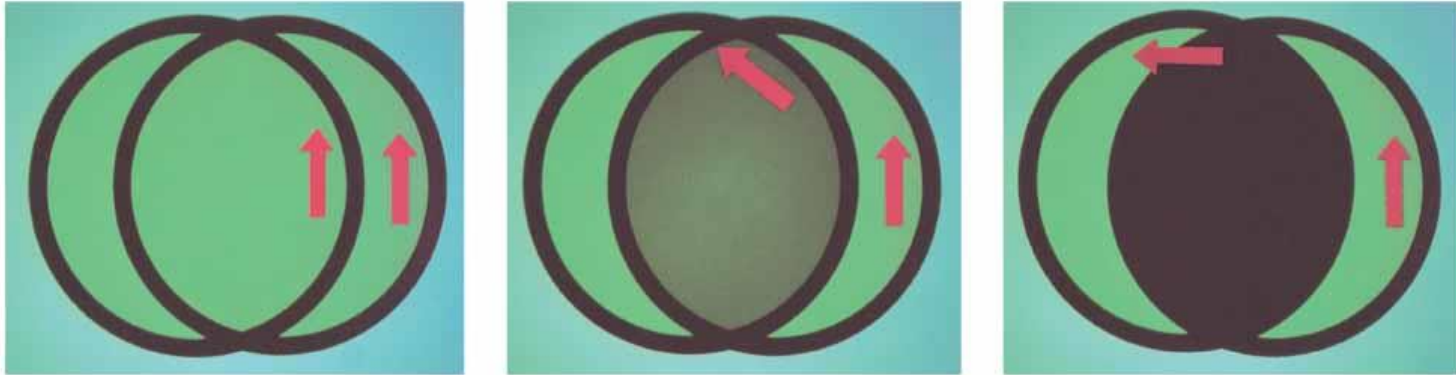
- The intensity of the polarized beam transmitted through the second polarizing sheet (the analyzer) varies as
 - $I = I_{max} \cos^2 \theta$
 - I_{max} is the intensity of the polarized wave incident on the analyzer
 - This is known as **Malus' law** and applies to any two polarizing materials whose transmission axes are at an angle of θ to each other

Intensity of a Polarized Beam, cont.



- The intensity of the transmitted beam is a maximum when the transmission axes are parallel
 - $\theta = 0$ or 180°
- The intensity is zero when the transmission axes are perpendicular to each other
 - This would cause complete absorption

Intensity of Polarized Light, Examples



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- On the left, the transmission axes are aligned and maximum intensity occurs
- In the middle, the axes are at 45° to each other and less intensity occurs
- On the right, the transmission axes are perpendicular and the light intensity is a minimum



Polarization by Reflection

- When an unpolarized light beam is reflected from a surface, the reflected light may be
 - Completely polarized
 - Partially polarized
 - Unpolarized
- It depends on the angle of incidence
 - If the angle is 0° , the reflected beam is unpolarized
 - For other angles, there is some degree of polarization
 - For one particular angle, the beam is completely polarized

Polarization by Reflection, cont.



- The angle of incidence for which the reflected beam is completely polarized is called the polarizing angle, θ_p
- **Brewster's law** relates the polarizing angle to the index of refraction for the material

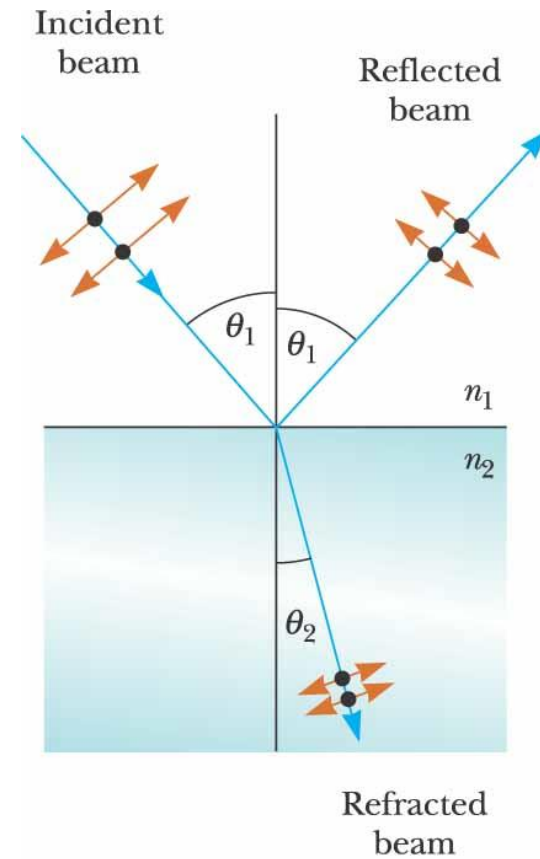
$$\tan \theta_p = \frac{n_2}{n_1}$$

- θ_p may also be called Brewster's angle

Polarization by Reflection, Partially Polarized Example



- Unpolarized light is incident on a reflecting surface
- The reflected beam is partially polarized
- The refracted beam is partially polarized

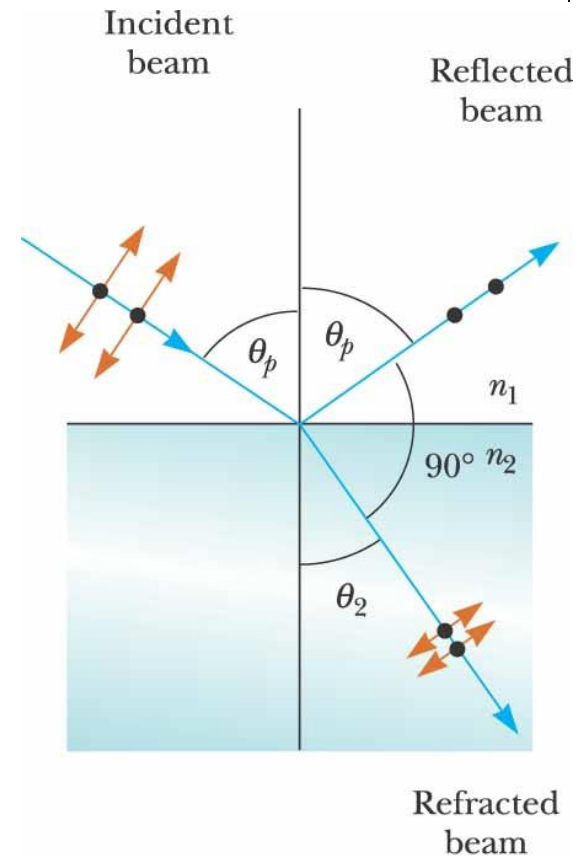


(a)

Polarization by Reflection, Completely Polarized Example



- Unpolarized light is incident on a reflecting surface
- The reflected beam is completely polarized
- The refracted beam is perpendicular to the reflected beam
- The angle of incidence is Brewster's angle



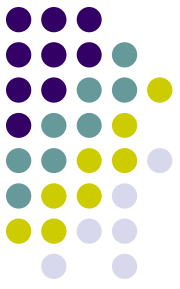
(b)

Polarization by Double Refraction

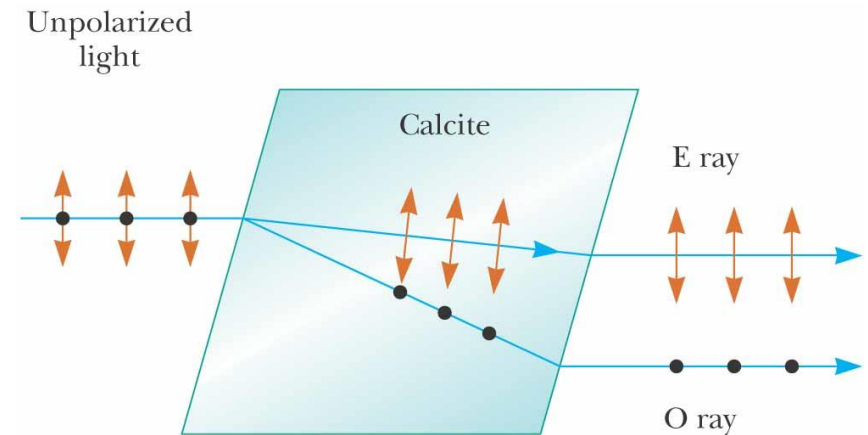


- In certain crystalline structures, the speed of light is not the same in all directions
- Such materials are characterized by two indices of refraction
- They are often called **double-refracting** or **birefringent** materials

Polarization by Double Refraction, cont.



- Unpolarized light splits into two plane-polarized rays
- The two rays are in mutual perpendicular directions
 - Indicated by the dots and arrows



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Polarization by Double Refraction, Rays

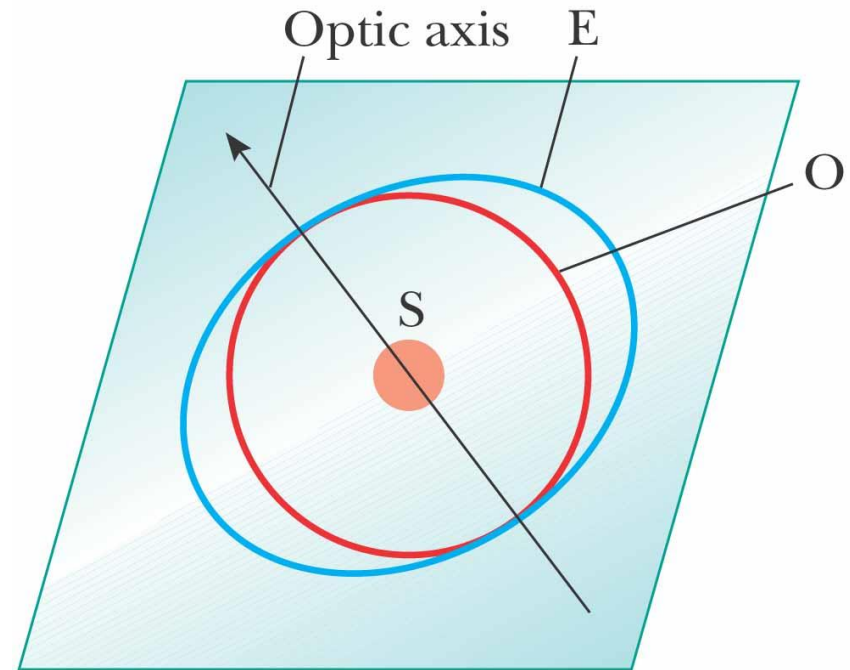


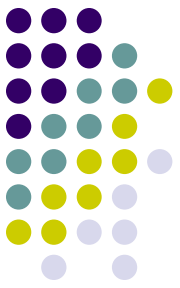
- The **ordinary (O) ray** is characterized by an index of refraction of n_o
 - This is the same in all directions
- The second ray is the ***extraordinary (E) ray*** which travels at different speeds in different directions
 - Characterized by an index of refraction of n_E that varies with the direction of propagation

Polarization by Double Refraction, Optic Axis



- There is one direction, called the **optic axis**, along which the ordinary and extraordinary rays have the same speed
 - $n_O = n_E$
- The difference in speeds for the two rays is a maximum in the direction perpendicular to the optic axis





Some Indices of Refraction

TABLE 38.1

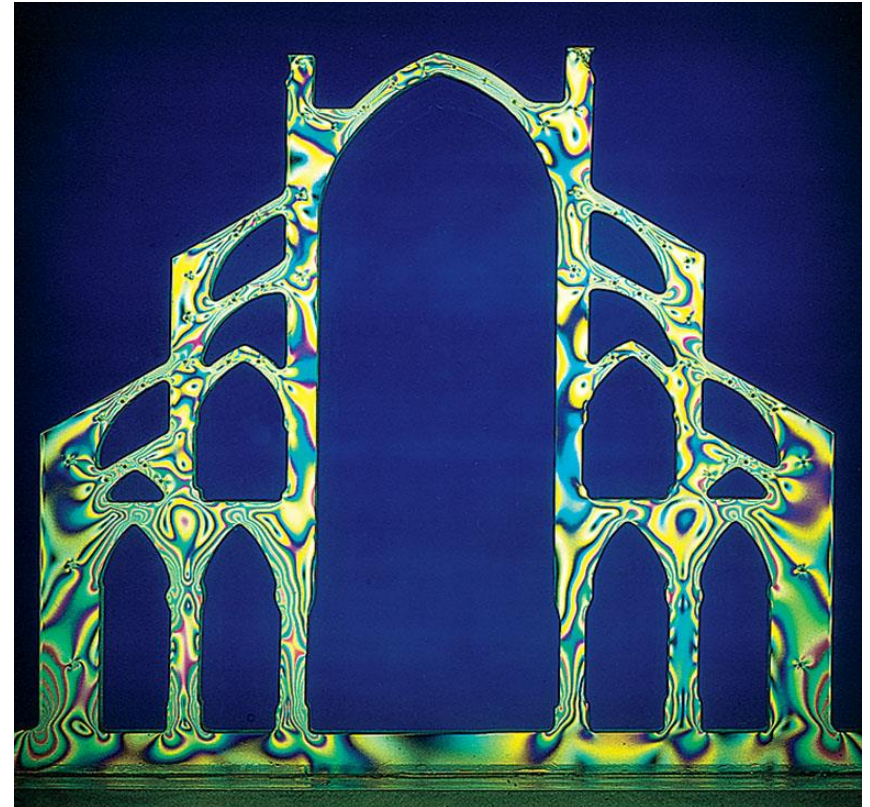
Indices of Refraction for Some Double-Refracting Crystals at a Wavelength of 589.3 nm

| Crystal | n_O | n_E | n_O/n_E |
|------------------------------------|-------|-------|-----------|
| Calcite (CaCO_3) | 1.658 | 1.486 | 1.116 |
| Quartz (SiO_2) | 1.544 | 1.553 | 0.994 |
| Sodium nitrate (NaNO_3) | 1.587 | 1.336 | 1.188 |
| Sodium sulfite (NaSO_3) | 1.565 | 1.515 | 1.033 |
| Zinc chloride (ZnCl_2) | 1.687 | 1.713 | 0.985 |
| Zinc sulfide (ZnS) | 2.356 | 2.378 | 0.991 |

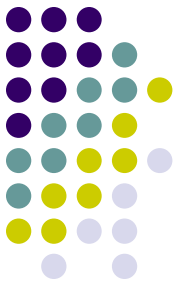
Optical Stress Analysis



- Some materials become birefringent when stressed
- When a material is stressed, a series of light and dark bands is observed
 - The light bands correspond to areas of greatest stress
- Optical stress analysis uses plastic models to test for regions of potential weaknesses



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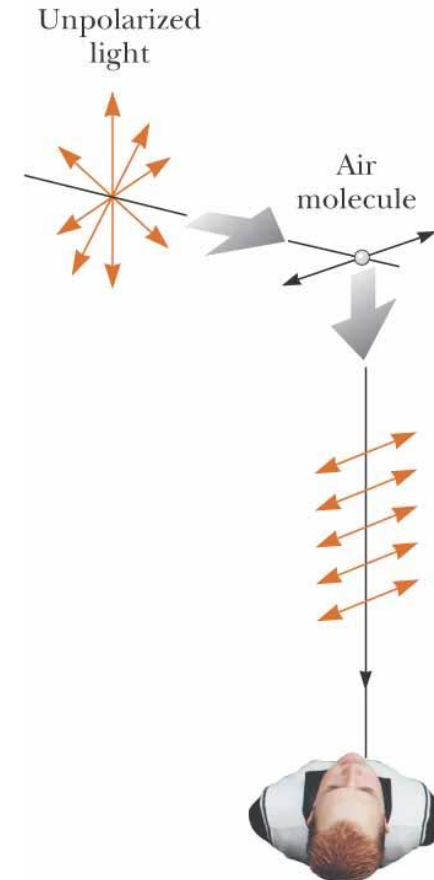
Polarization by Scattering

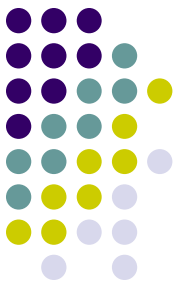
- When light is incident on any material, the electrons in the material can absorb and reradiate part of the light
 - This process is called **scattering**
- An example of scattering is the sunlight reaching an observer on the Earth being partially polarized

Polarization by Scattering, cont.



- The horizontal part of the electric field vector in the incident wave causes the charges to vibrate horizontally
- The vertical part of the vector simultaneously causes them to vibrate vertically
- If the observer looks straight up, he sees light that is completely polarized in the horizontal direction

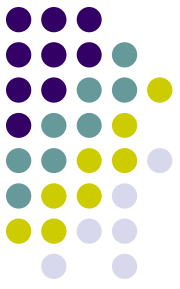




Scattering, cont.

- Short wavelengths (violet) are scattered more efficiently than long wavelengths (red)
- When sunlight is scattered by gas molecules in the air, the violet is scattered more intensely than the red
- When you look up, you see blue
 - Your eyes are more sensitive to blue, so you see blue instead of violet
- At sunrise or sunset, much of the blue is scattered away, leaving the light at the red end of the spectrum

Optical Activity



- Certain materials display the property of **optical activity**
 - A material is said to be optically active if it rotates the plane of polarization of any light transmitted through it
 - Molecular asymmetry determines whether a material is optically active