Chapter 30

Sources of the Magnetic Field

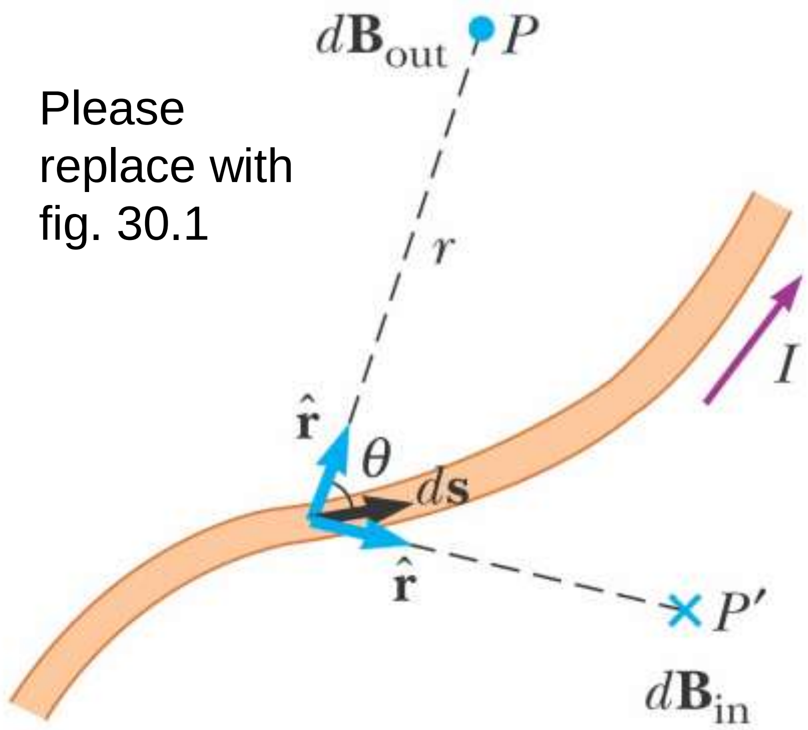
Biot-Savart Law – Introduction

- Biot and Savart conducted experiments on the force exerted by an electric current on a nearby magnet
- They arrived at a mathematical expression that gives the magnetic field at some point in space due to a current

Biot-Savart Law – Set-Up

- The magnetic field is $d\vec{\mathbf{B}}$ at some point P
- The length element is $d\vec{\mathbf{s}}$
- The wire is carrying a steady current of I

Please replace with fig. 30.1



Biot-Savart Law – Observations

- The vector $d\vec{\mathbf{B}}$ is perpendicular to both $d\vec{\mathbf{s}}$ and to the unit vector $\hat{\mathbf{r}}$ directed from $d\vec{\mathbf{s}}$ toward P
- The magnitude of $d\vec{\mathbf{B}}$ is inversely proportional to r^2, where r is the distance from $d\vec{\mathbf{s}}$ to P

Biot-Savart Law – Observations, cont

- The magnitude of $d\vec{\mathbf{B}}$ is proportional to the current and to the magnitude ds of the length element $d\vec{\mathbf{s}}$
- The magnitude of $d\vec{\mathbf{B}}$ is proportional to sin θ, where θ is the angle between the vectors $d\vec{\mathbf{s}}$ and $\hat{\mathbf{r}}$

Biot-Savart Law – Equation

- The observations are summarized in the mathematical equation called the **Biot-Savart law**:

$$d\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \frac{I \, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

- The magnetic field described by the law is the field *due to* the current-carrying conductor
  - Don’t confuse this field with a field *external* to the conductor

Permeability of Free Space

- The constant μ_o is called the **permeability of free space**
- $\mu_o = 4\pi \times 10^{-7}\ T\cdot m / A$

Total Magnetic Field

- $d\vec{\mathbf{B}}$ is the field created by the current in the length segment ds
- To find the total field, sum up the contributions from all the current elements I $d\vec{\mathbf{s}}$

$$\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi} \int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

  - The integral is over the entire current distribution

Biot-Savart Law – Final Notes

- The law is also valid for a current consisting of charges flowing through space
- $d\vec{\mathbf{s}}$ represents the length of a small segment of space in which the charges flow
  - For example, this could apply to the electron beam in a TV set

$\vec{B}$ Compared to $\vec{E}$

- Distance
  - The magnitude of the magnetic field varies as the inverse square of the distance from the source
  - The electric field due to a point charge also varies as the inverse square of the distance from the charge

$\vec{B}$ Compared to $\vec{E}$, 2

- Direction
  - The electric field created by a point charge is radial in direction
  - The magnetic field created by a current element is perpendicular to both the length element $d\vec{\mathbf{s}}$ and the unit vector $\hat{\mathbf{r}}$

$\vec{B}$ Compared to $\vec{E}$, 3

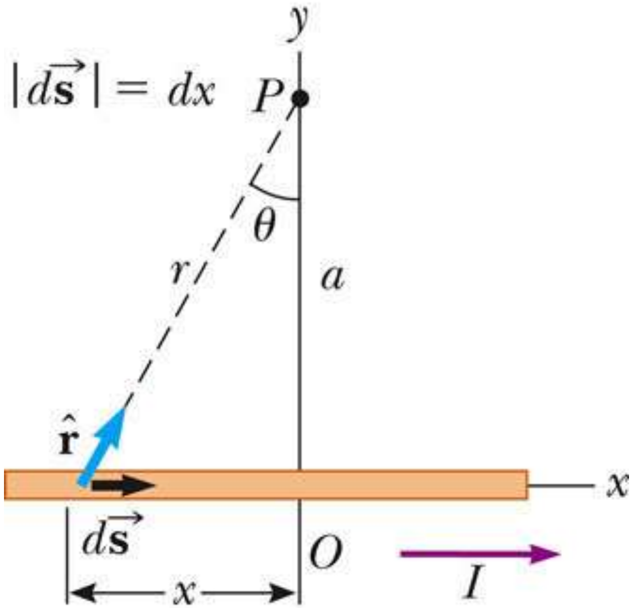
- Source
  - An electric field is established by an isolated electric charge
  - The current element that produces a magnetic field must be part of an extended current distribution
    - Therefore you must integrate over the entire current distribution

$\vec{B}$ for a Long, Straight Conductor

- The thin, straight wire is carrying a constant current
- $d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = (dx \sin\theta)\hat{\mathbf{k}}$
- Integrating over all the current elements gives

$$B = -\frac{\mu_o I}{4\pi a}\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta$$

$$= \frac{\mu_o I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

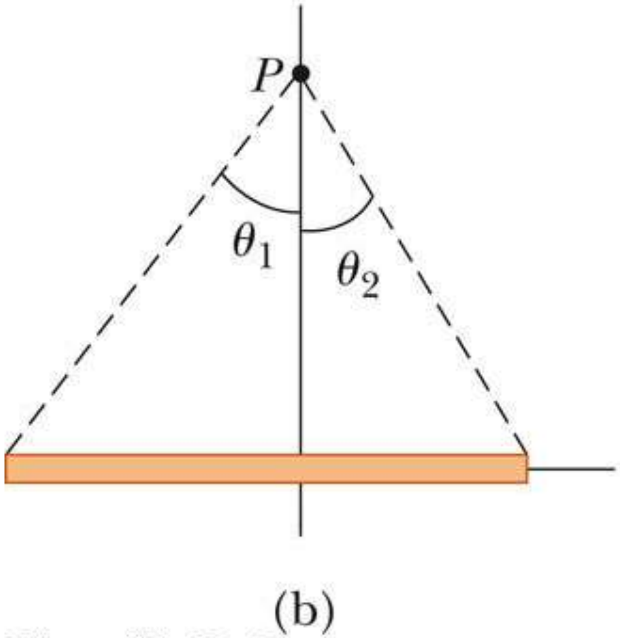


(a)

$\vec{B}$ for a Long, Straight Conductor, Special Case

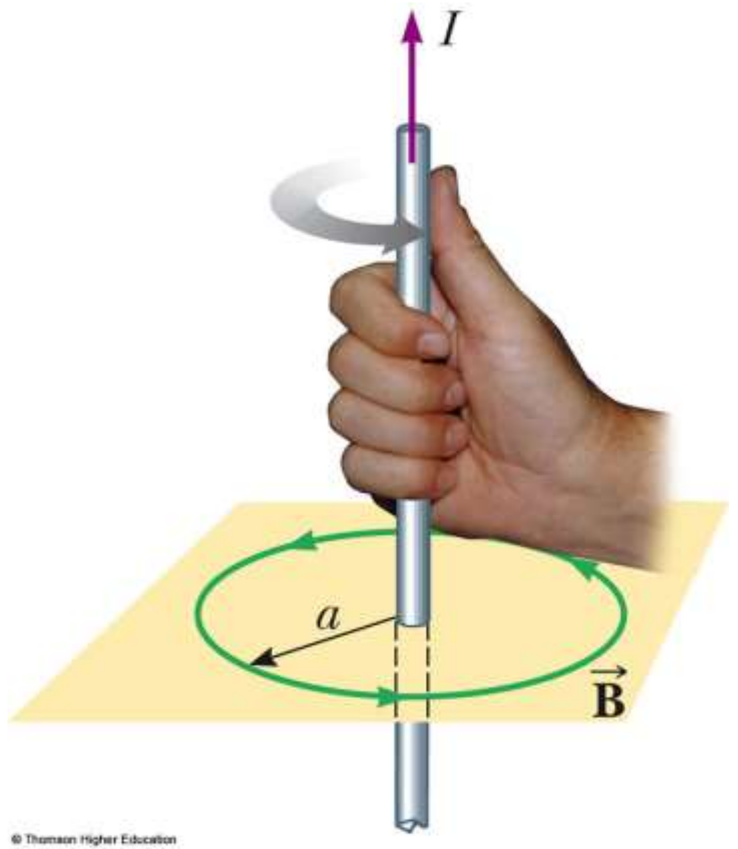
- If the conductor is an infinitely long, straight wire, $\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$
- The field becomes

$$B = \frac{\mu_o I}{2\pi a}$$



$\vec{B}$ for a Long, Straight Conductor, Direction

- The magnetic field lines are circles concentric with the wire
- The field lines lie in planes perpendicular to to wire
- The magnitude of the field is constant on any circle of radius *a*
- The right-hand rule for determining the direction of the field is shown

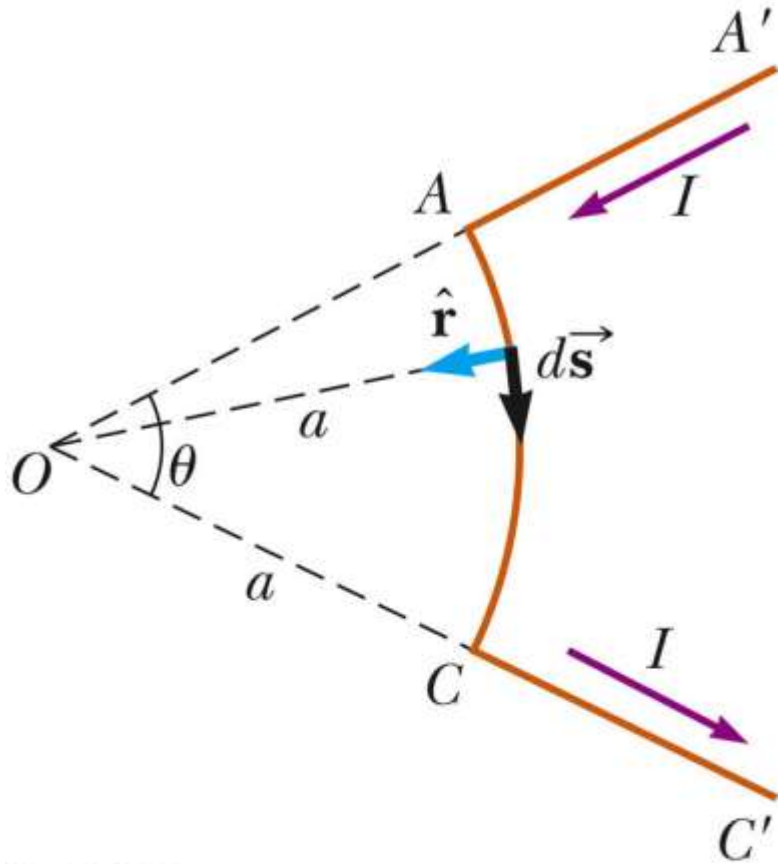


$\vec{B}$ for a Curved Wire Segment

- Find the field at point *O* due to the wire segment
- *I* and *R* are constants

$$B = \frac{\mu_o I}{4\pi R}\theta$$

  - θ will be in radians



$\vec{B}$ for a Circular Loop of Wire

- Consider the previous result, with a full circle
  - $\theta = 2\pi$

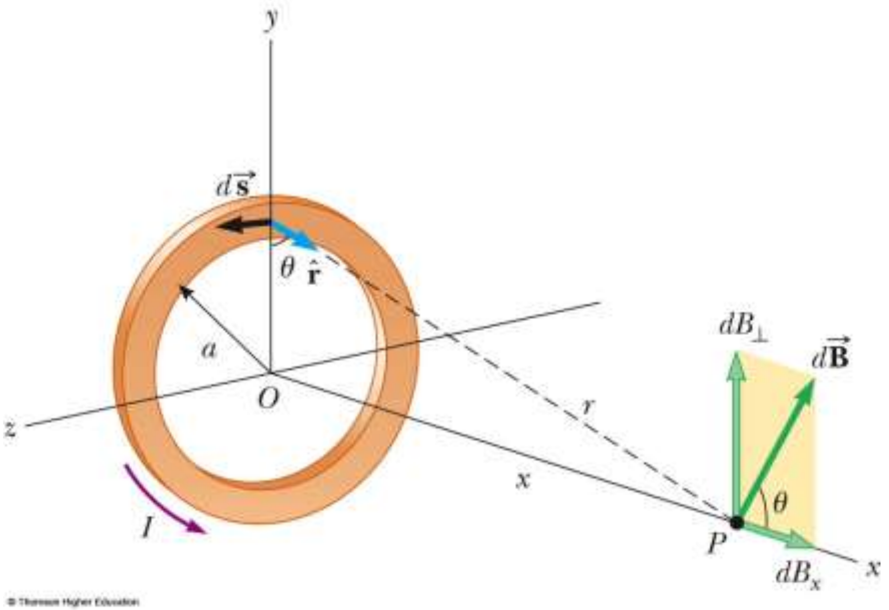
$$B = \frac{\mu_o I}{4\pi a}\theta = \frac{\mu_o I}{4\pi a}2\pi = \frac{\mu_o I}{2a}$$

- This is the field at the *center* of the loop

$\vec{B}$ for a Circular Current Loop

- The loop has a radius of R and carries a steady current of I
- Find the field at point P

$$B_x = \frac{\mu_o I a^2}{2\left(a^2 + x^2\right)^{3/2}}$$



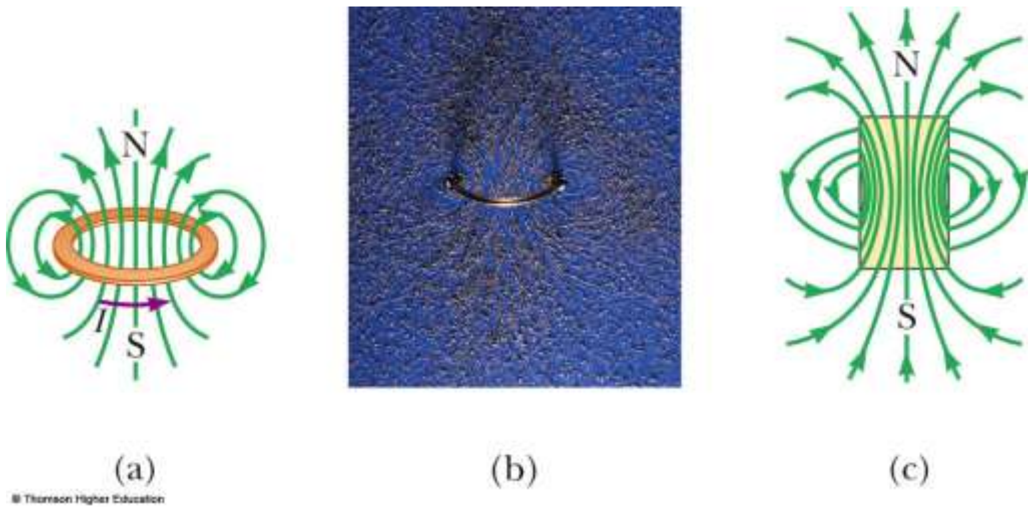
Comparison of Loops

- Consider the field at the center of the current loop
- At this special point, $x = 0$
- Then,

$$B_x = \frac{\mu_o I a^2}{2\left(a^2 + x^2\right)^{3/2}} = \frac{\mu_o I}{2a}$$

  - This is exactly the same result as from the curved wire

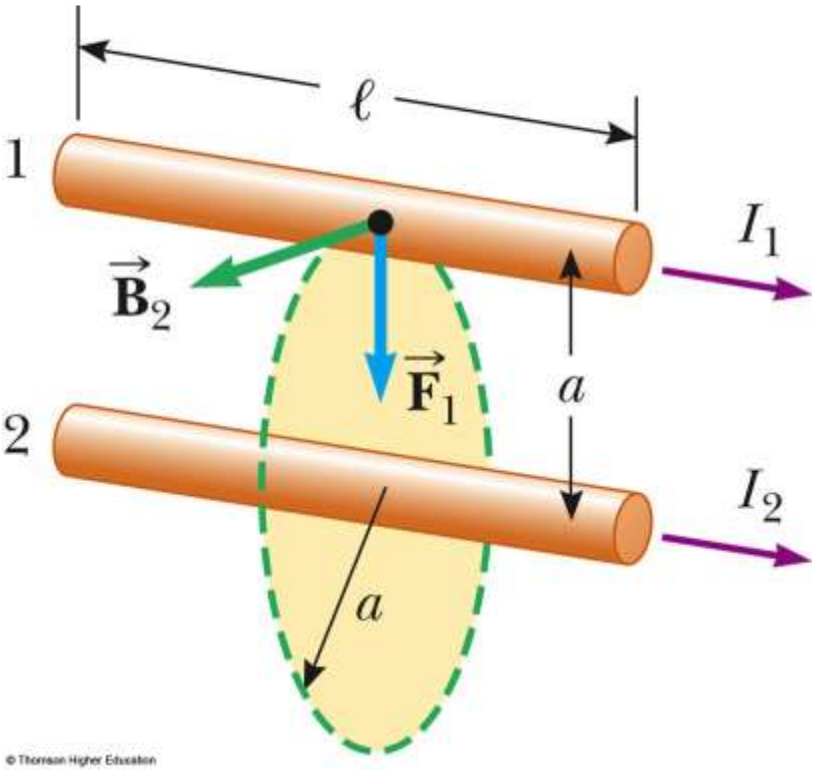
Magnetic Field Lines for a Loop



- Figure (a) shows the magnetic field lines surrounding a current loop
- Figure (b) shows the field lines in the iron filings
- Figure (c) compares the field lines to that of a bar magnet

Magnetic Force Between Two Parallel Conductors

- Two parallel wires each carry a steady current
- The field $\vec{\mathbf{B}}_2$ due to the current in wire 2 exerts a force on wire 1 of $F_1 = I_1 \ell B_2$



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Magnetic Force Between Two Parallel Conductors, cont.

- Substituting the equation for $\vec{\mathbf{B}}_2$ gives

$$F_1 = \frac{\mu_o I_1 I_2}{2\pi a}\ell$$

  - Parallel conductors carrying currents in the same direction attract each other
  - Parallel conductors carrying current in opposite directions repel each other

Magnetic Force Between Two Parallel Conductors, final

- The result is often expressed as the magnetic force between the two wires, F_B
- This can also be given as the force per unit length:

$$\frac{F_B}{\ell} = \frac{\mu_o I_1 I_2}{2\pi a}$$

Definition of the Ampere

- The force between two parallel wires can be used to define the ampere
- When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A

Definition of the Coulomb

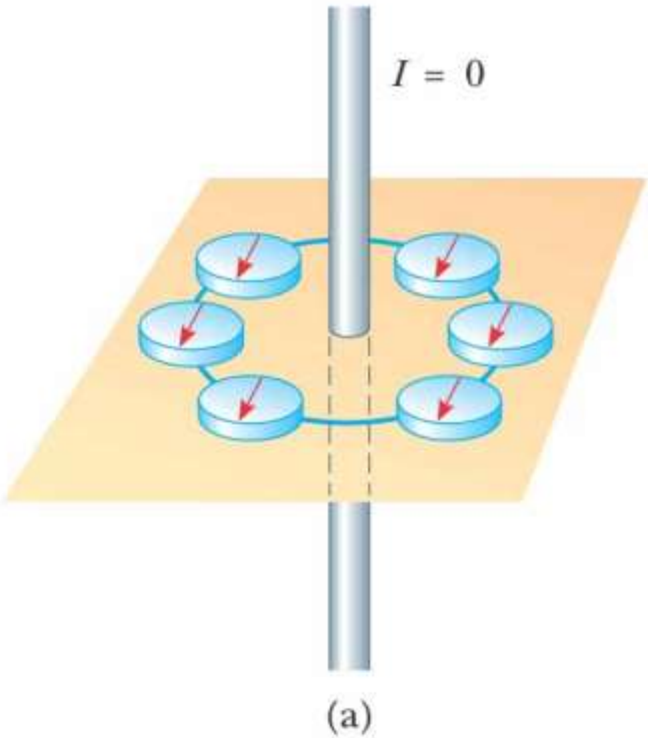
- The SI unit of charge, the coulomb, is defined in terms of the ampere
- When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C

Andre-Marie Ampère

- 1775 – 1836
- French physicist
- Created with the discovery of electromagnetism
  - The relationship between electric current and magnetic fields
- Also worked in mathematics

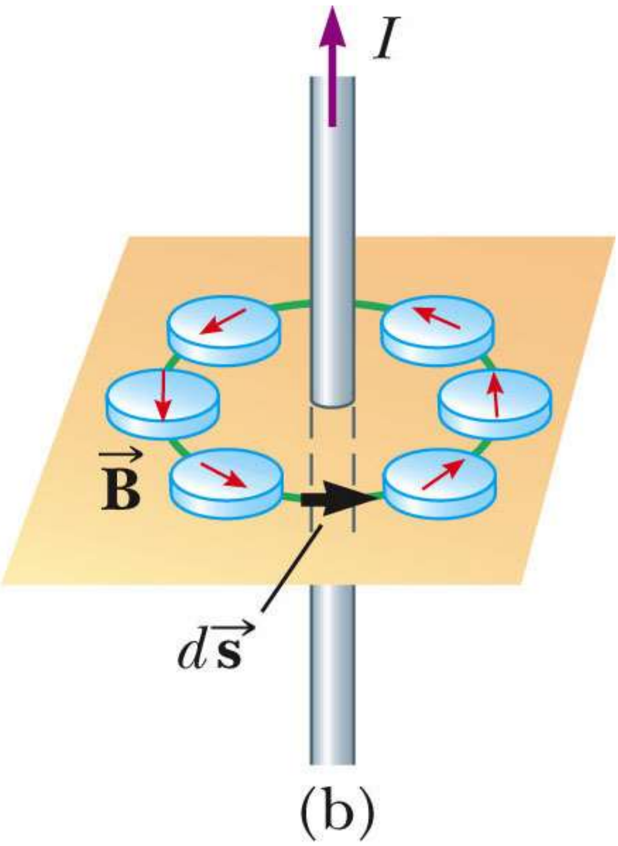
Magnetic Field of a Wire

- A compass can be used to detect the magnetic field
- When there is no current in the wire, there is no field due to the current
- The compass needles all point toward the Earth's north pole
  - Due to the Earth's magnetic field



Magnetic Field of a Wire, 2

- Here the wire carries a strong current
- The compass needles deflect in a direction tangent to the circle
- This shows the direction of the magnetic field produced by the wire
- Use the active figure to vary the current



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Magnetic Field of a Wire, 3

- The circular magnetic field around the wire is shown by the iron filings

Ampere's Law

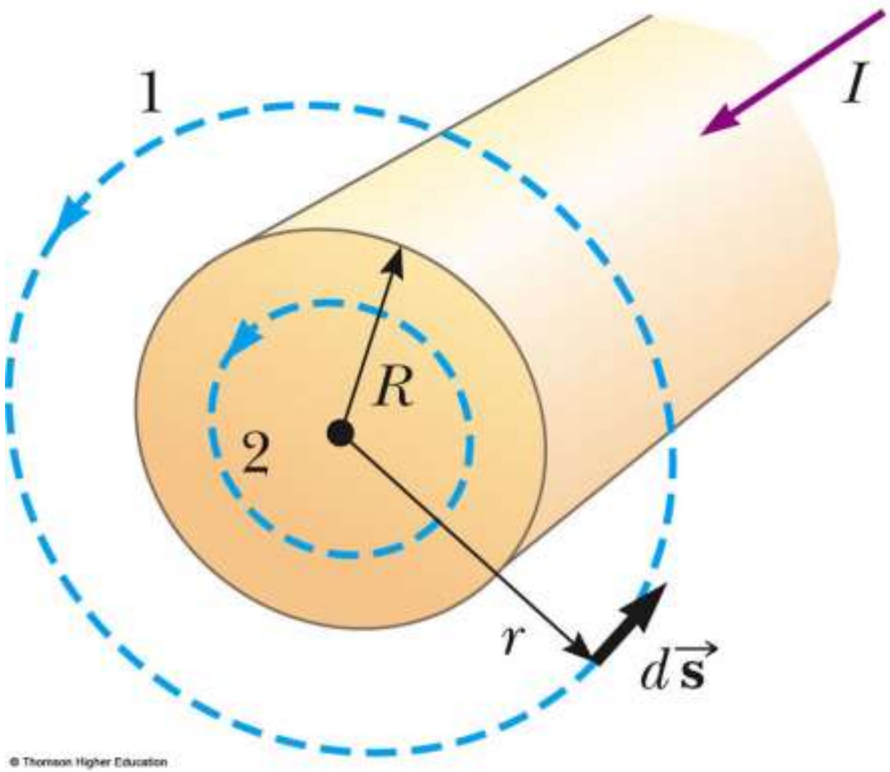
- The product of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ can be evaluated for small length elements $d\vec{\mathbf{s}}$ on the circular path defined by the compass needles for the long straight wire
- Ampere's law states that the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed path equals $\mu_o I$ where I is the total steady current passing through any surface bounded by the closed path: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o I$

Ampere's Law, cont.

- Ampere's law describes the creation of magnetic fields by all continuous current configurations
  - Most useful for this course if the current configuration has a high degree of symmetry
- Put the thumb of your right hand in the direction of the current through the amperian loop and your fingers curl in the direction you should integrate around the loop

Field Due to a Long Straight Wire – From Ampere's Law

- Want to calculate the magnetic field at a distance *r* from the center of a wire carrying a steady current I
- The current is uniformly distributed through the cross section of the wire



Field Due to a Long Straight Wire – Results From Ampere's Law

- Outside of the wire, $r > R$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_o I \quad \rightarrow \quad B = \frac{\mu_o I}{2\pi r}$$

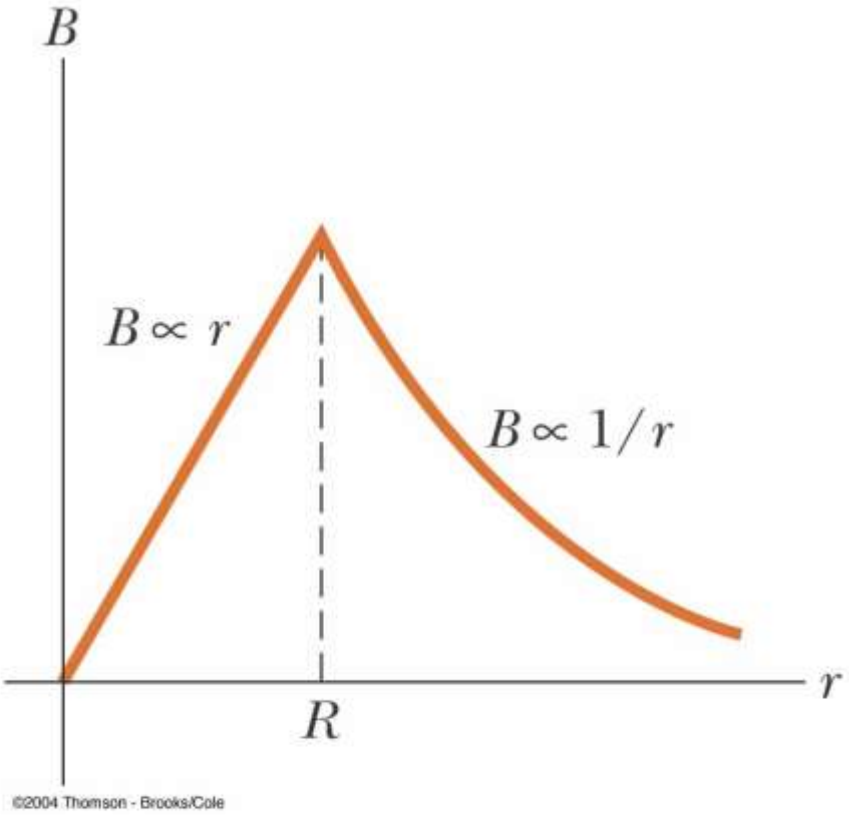
- Inside the wire, we need I', the current inside the amperian circle

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_o I' \quad \rightarrow \quad I' = \frac{r^2}{R^2} I$$

$$B = \left(\frac{\mu_o I}{2\pi R^2} \right) r$$

Field Due to a Long Straight Wire – Results Summary

- The field is proportional to *r* inside the wire
- The field varies as 1/*r* outside the wire
- Both equations are equal at *r* = *R*

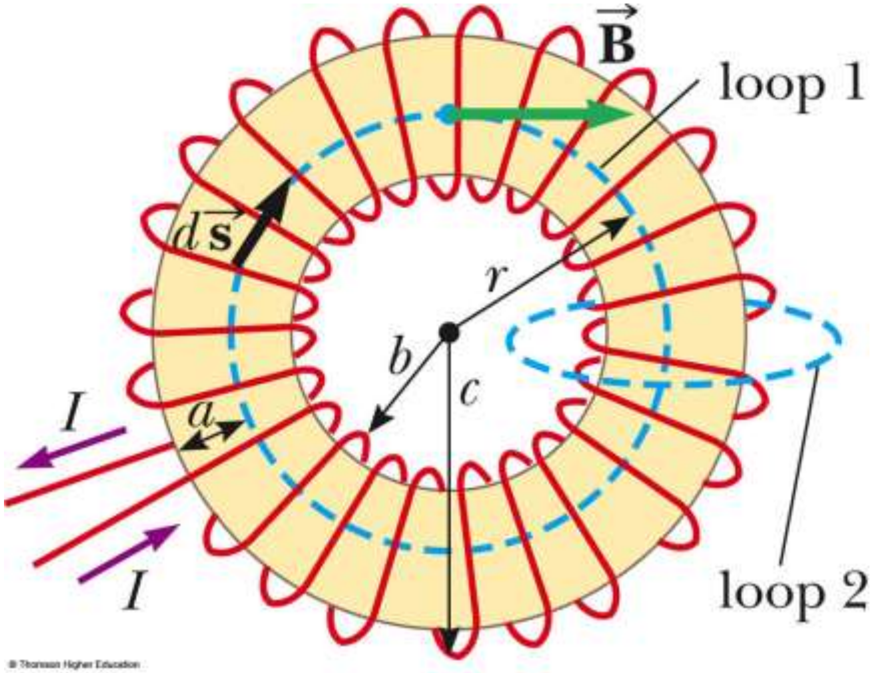


Magnetic Field of a Toroid

- Find the field at a point at distance *r* from the center of the toroid
- The toroid has *N* turns of wire

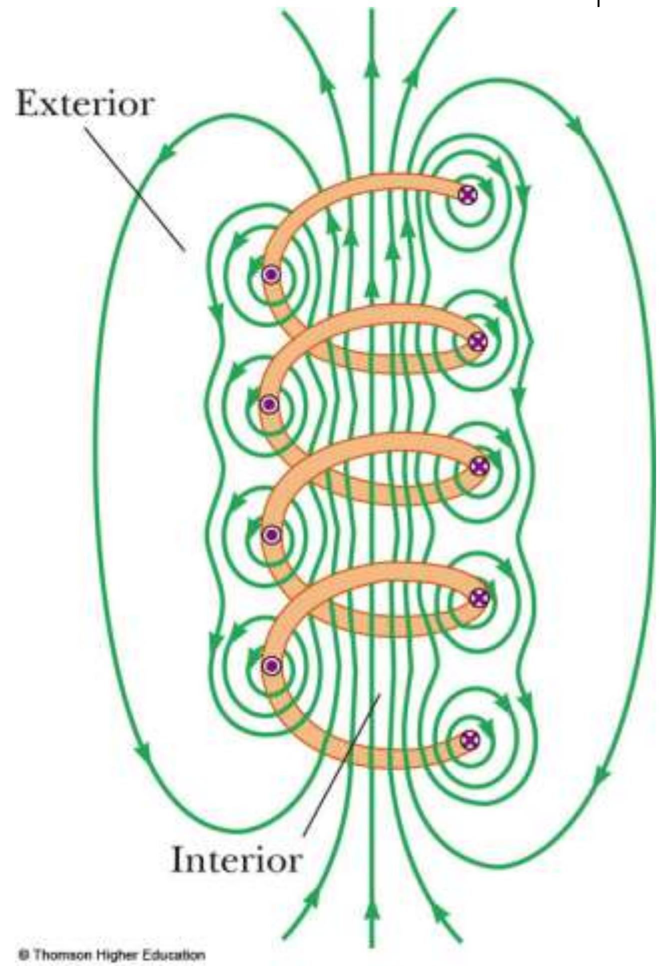
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_o N I$$

$$B = \frac{\mu_o N I}{2\pi r}$$



Magnetic Field of a Solenoid

- A **solenoid** is a long wire wound in the form of a helix
- A reasonably uniform magnetic field can be produced in the space surrounded by the turns of the wire
  - The ***interior*** of the solenoid

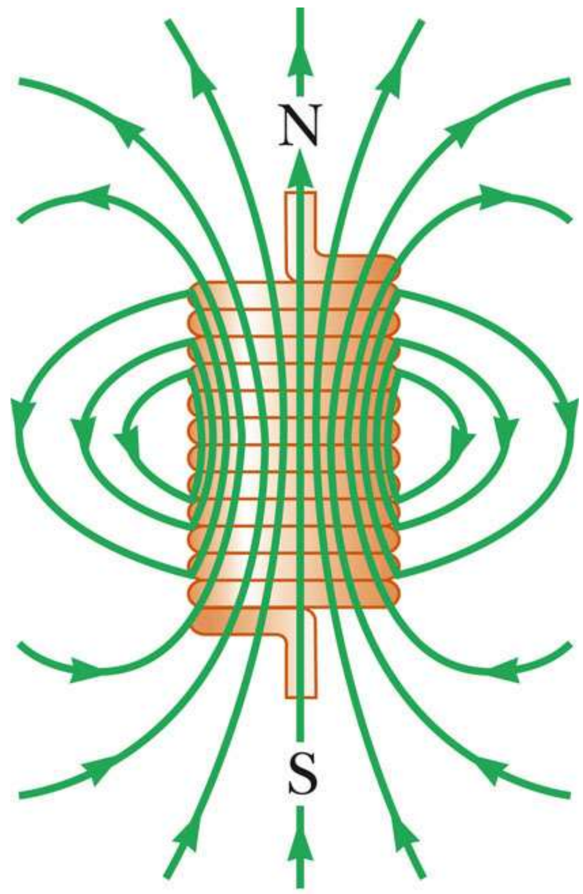


Magnetic Field of a Solenoid, Description

- The field lines in the interior are
  - nearly parallel to each other
  - uniformly distributed
  - close together
- This indicates the field is strong and almost uniform

Magnetic Field of a Tightly Wound Solenoid

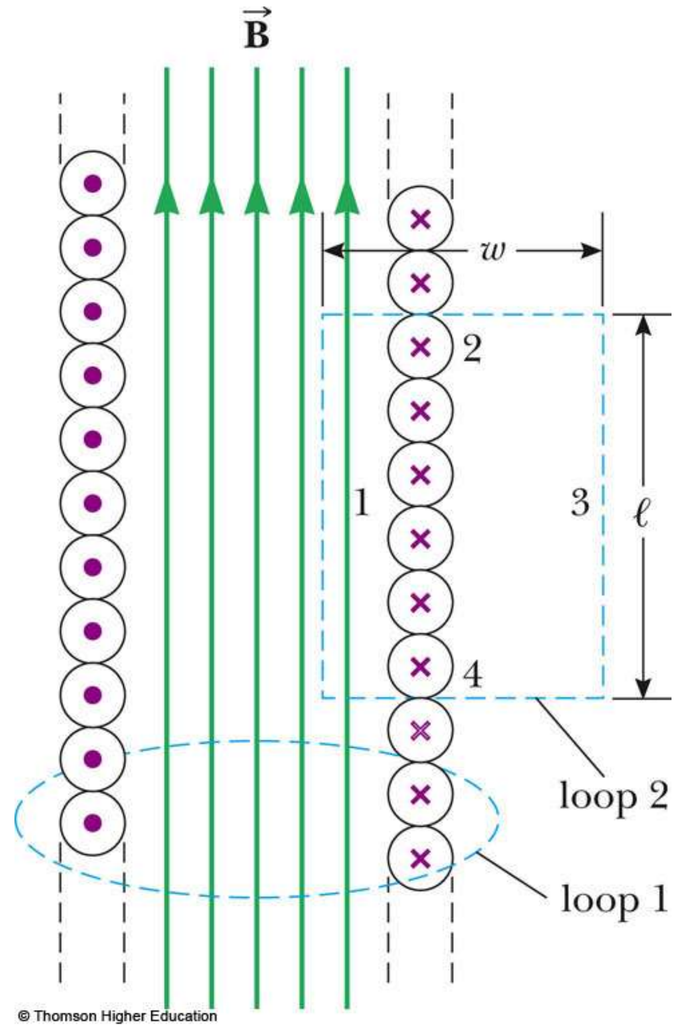
- The field distribution is similar to that of a bar magnet
- As the length of the solenoid increases
  - the interior field becomes more uniform
  - the exterior field becomes weaker



(a)

Ideal Solenoid – Characteristics

- An *ideal solenoid* is approached when:
  - the turns are closely spaced
  - the length is much greater than the radius of the turns



Ampere's Law Applied to a Solenoid

- Ampere's law can be used to find the interior magnetic field of the solenoid
- Consider a rectangle with side ℓ parallel to the interior field and side w perpendicular to the field
  - This is loop 2 in the diagram
- The side of length ℓ inside the solenoid contributes to the field
  - This is side 1 in the diagram

Ampere's Law Applied to a Solenoid, cont.

- Applying Ampere's Law gives

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{path1} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \int_{path1} ds = B\ell$$

- The total current through the rectangular path equals the current through each turn multiplied by the number of turns

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_o NI$$

Magnetic Field of a Solenoid, final

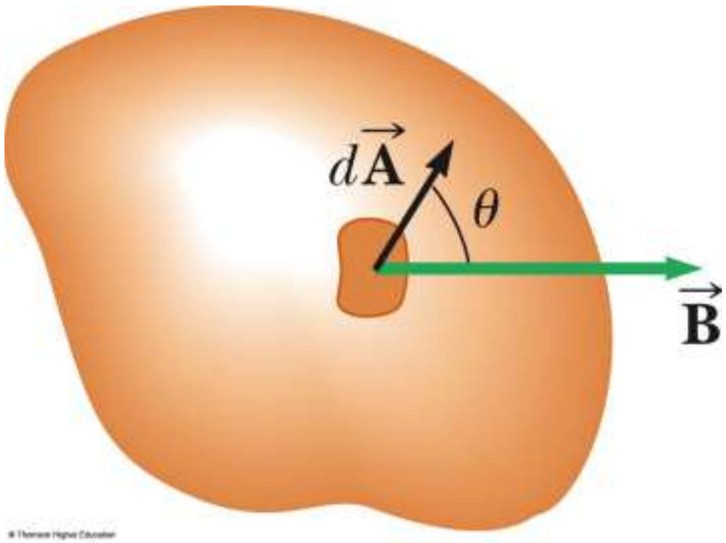
- Solving Ampere's law for the magnetic field is

$$B = \mu_o \frac{N}{\ell} I = \mu_o n I$$

  - $n = N / \ell$ is the number of turns per unit length
- This is valid only at points near the center of a very long solenoid

Magnetic Flux

- The magnetic flux associated with a magnetic field is defined in a way similar to electric flux
- Consider an area element *dA* on an arbitrarily shaped surface



Magnetic Flux, cont.

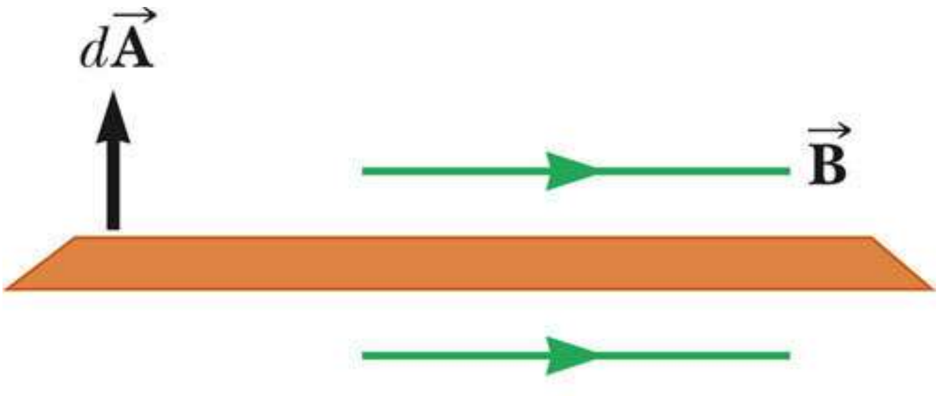
- The magnetic field in this element is $\vec{\mathbf{B}}$
- $d\vec{\mathbf{A}}$ is a vector that is perpendicular to the surface
- $d\vec{\mathbf{A}}$ has a magnitude equal to the area *dA*
- The magnetic flux Φ_B is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

- The unit of magnetic flux is T·m^2 = Wb
  - Wb is a *weber*

Magnetic Flux Through a Plane, 1

- A special case is when a plane of area *A* makes an angle θ with $d\vec{\mathbf{A}}$
- The magnetic flux is $\Phi_B = BA\cos\theta$
- In this case, the field is parallel to the plane and $\Phi = 0$



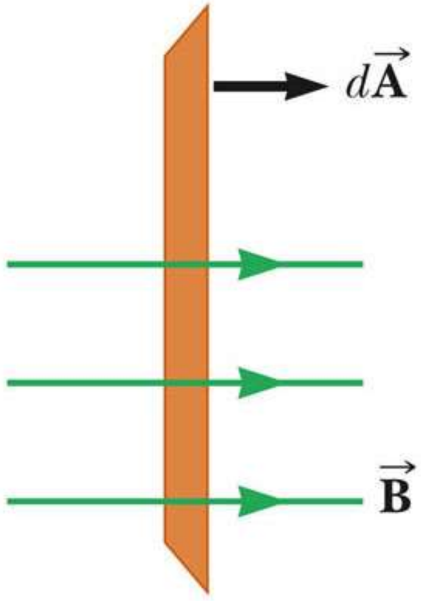
(a)

PLAY ACTIVE FIGURE

Magnetic Flux Through A Plane, 2

- The magnetic flux is $\Phi_B = BA \cos \theta$
- In this case, the field is perpendicular to the plane and

  $\Phi = BA$
- This will be the maximum value of the flux
- Use the active figure to investigate different angles



(b)

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Gauss’ Law in Magnetism

- Magnetic fields do not begin or end at any point
  - The number of lines entering a surface equals the number of lines leaving the surface
- **Gauss’ law in magnetism** says the magnetic flux through any closed surface is always zero:

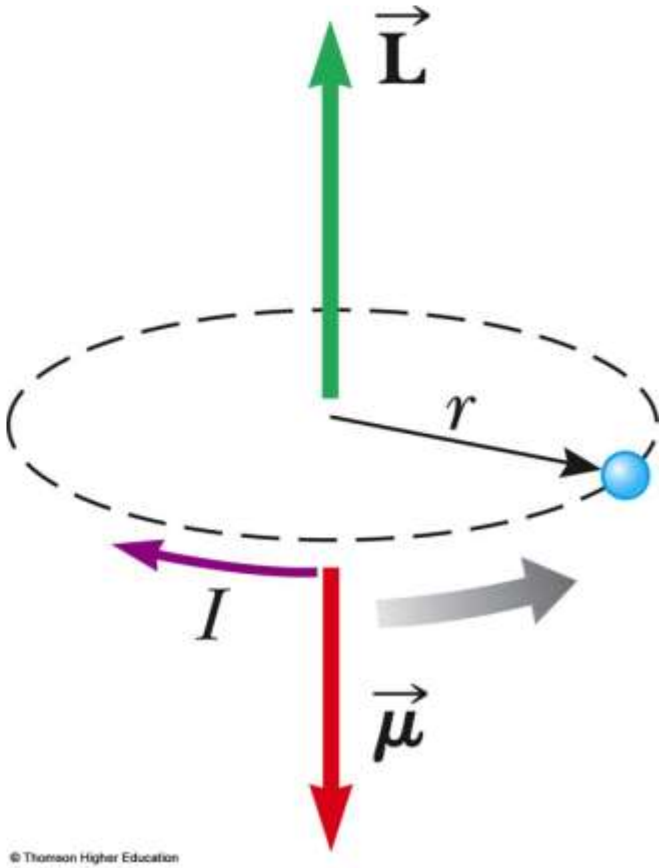
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

Magnetic Moments

- In general, any current loop has a magnetic field and thus has a magnetic dipole moment
- This includes atomic-level current loops described in some models of the atom
- This will help explain why some materials exhibit strong magnetic properties

Magnetic Moments – Classical Atom

- The electrons move in circular orbits
- The orbiting electron constitutes a tiny current loop
- The magnetic moment of the electron is associated with this orbital motion
- $\vec{\mathbf{L}}$ is the angular momentum
- $\vec{\mu}$ is magnetic moment



Magnetic Moments – Classical Atom, 2

- This model assumes the electron moves
  - with constant speed *v*
  - in a circular orbit of radius *r*
  - travels a distance $2\pi r$ in a time interval *T*
- The current associated with this orbiting electron is

$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

Magnetic Moments – Classical Atom, 3

- The magnetic moment is $\mu = I\,A = \frac{1}{2}evr$
- The magnetic moment can also be expressed in terms of the angular momentum

$$\mu = \left(\frac{e}{2m_e}\right)L$$

Magnetic Moments – Classical Atom, final

- The magnetic moment of the electron is proportional to its orbital angular momentum
  - The vectors $\vec{\mathbf{L}}$ and $\vec{\mu}$ point in *opposite* directions
  - Because the electron is negatively charged
- Quantum physics indicates that angular momentum is quantized

Magnetic Moments of Multiple Electrons

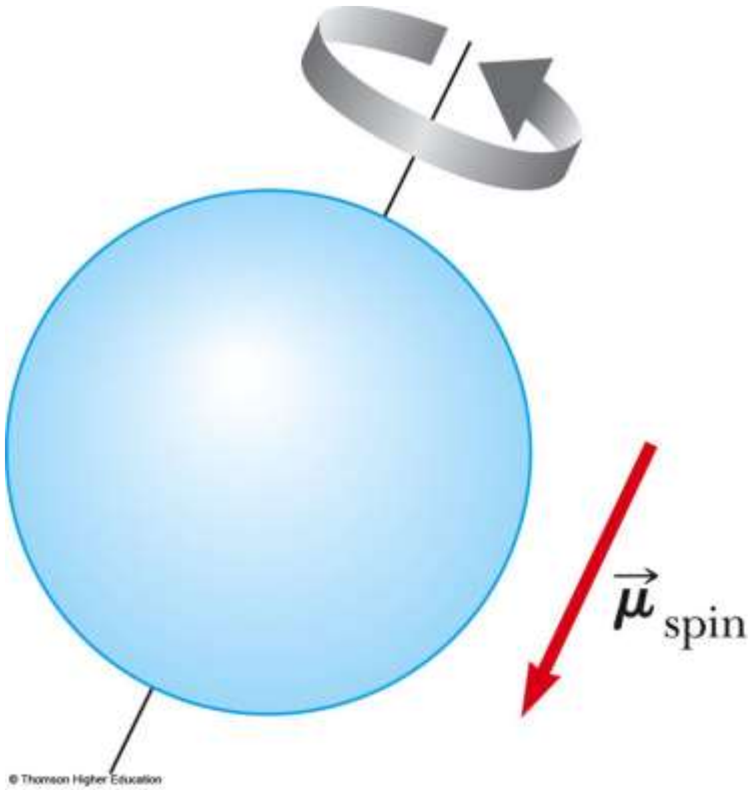
- In most substances, the magnetic moment of one electron is canceled by that of another electron orbiting in the same direction
- The net result is that the magnetic effect produced by the orbital motion of the electrons is either zero or very small

Electron Spin

- Electrons (and other particles) have an intrinsic property called **spin** that also contributes to their magnetic moment
  - The electron is not physically spinning
  - It has an intrinsic angular momentum as if it were spinning
  - Spin angular momentum is actually a relativistic effect

Electron Spin, cont.

- The classical model of electron spin is the electron spinning on its axis
- The magnitude of the spin angular momentum is
  $$S = \frac{\sqrt{3}}{2}\hbar$$
  - $\hbar$ is Planck's constant



Electron Spin and Magnetic Moment

- The magnetic moment characteristically associated with the spin of an electron has the value

$$\mu_{\text{spin}} = \frac{e\hbar}{2m_e}$$

- This combination of constants is called the **Bohr magneton** $\mu_{\mathbf{B}}$ = 9.27 x 10^{-24} J/T

Electron Magnetic Moment, final

- The total magnetic moment of an atom is the vector sum of the orbital and spin magnetic moments
- Some examples are given in the table at right
- The magnetic moment of a proton or neutron is much smaller than that of an electron and can usually be neglected

TABLE 30.1

Magnetic Moments of Some Atoms and Ions

Atom or Ion	Magnetic Moment (10^{-24} J/T)
H	9.27
He	0
Ne	0
Ce^{3+}	19.8
Yb^{3+}	37.1

Ferromagnetism

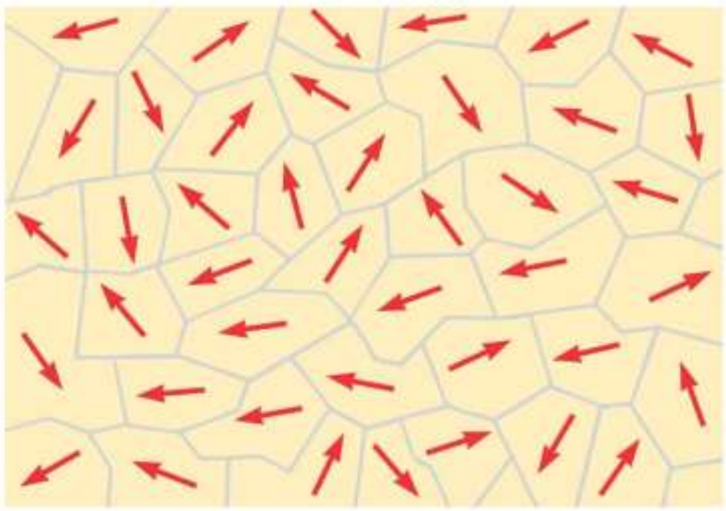
- Some substances exhibit strong magnetic effects called ferromagnetism
- Some examples of ferromagnetic materials are:
  - iron
  - cobalt
  - nickel
  - gadolinium
  - dysprosium
- They contain permanent atomic magnetic moments that tend to align parallel to each other even in a weak external magnetic field

Domains

- All ferromagnetic materials are made up of microscopic regions called **domains**
  - The domain is an area within which all magnetic moments are aligned
- The boundaries between various domains having different orientations are called **domain walls**

Domains, Unmagnetized Material

- The magnetic moments in the domains are randomly aligned
- The net magnetic moment is zero



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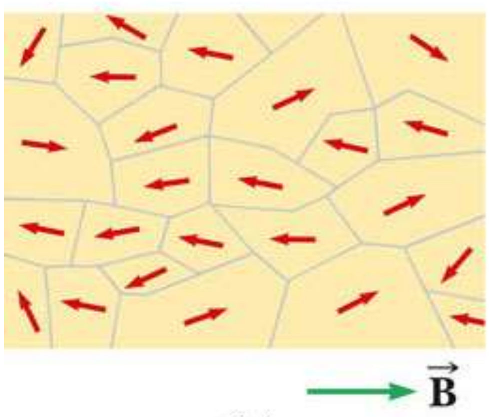
Domains, External Field Applied

- A sample is placed in an external magnetic field
- The size of the domains with magnetic moments aligned with the field grows
- The sample is magnetized



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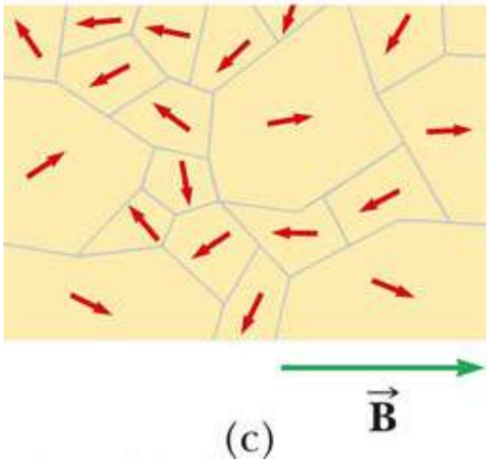
Domains, External Field Applied, cont.

- The material is placed in a stronger field
- The domains not aligned with the field become very small
- When the external field is removed, the material may retain a net magnetization in the direction of the original field



Curie Temperature

- The **Curie temperature** is the critical temperature above which a ferromagnetic material loses its residual magnetism
  - The material will become paramagnetic
- Above the Curie temperature, the thermal agitation is great enough to cause a random orientation of the moments

Table of Some Curie Temperatures

TABLE 30.2

Curie Temperatures for Several Ferromagnetic Substances

Substance	T_{Curie} (K)
Iron	1 043
Cobalt	1 394
Nickel	631
Gadolinium	317
Fe_2O_3	893

Paramagnetism

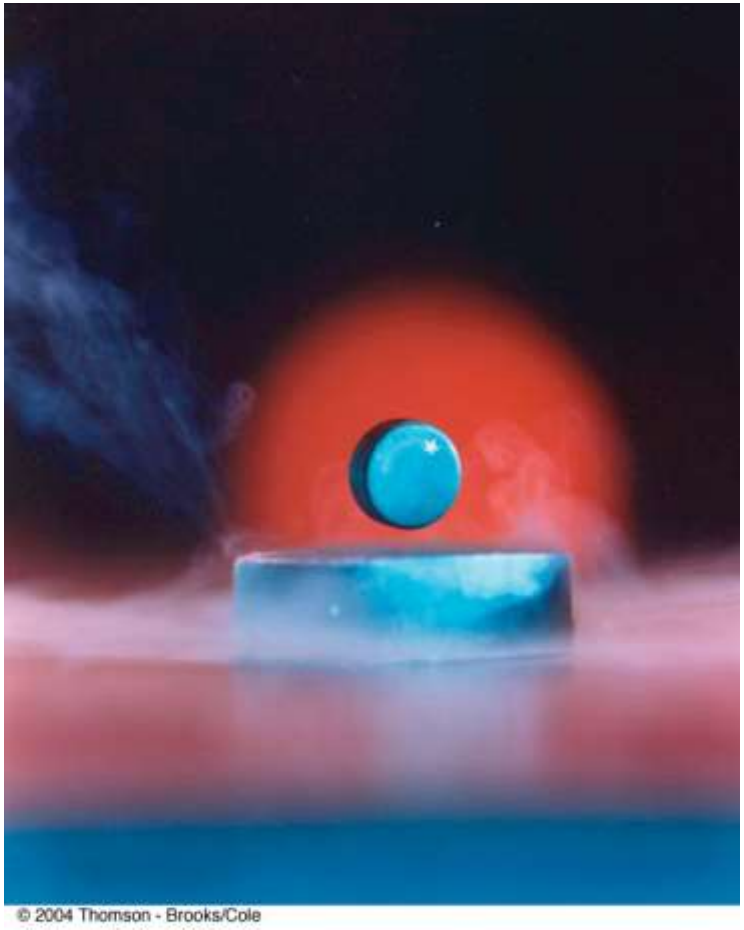
- Paramagnetic substances have small but positive magnetism
- It results from the presence of atoms that have permanent magnetic moments
  - These moments interact weakly with each other
- When placed in an external magnetic field, its atomic moments tend to line up with the field
  - The alignment process competes with thermal motion which randomizes the moment orientations

Diamagnetism

- When an external magnetic field is applied to a diamagnetic substance, a weak magnetic moment is induced in the direction opposite the applied field
- Diamagnetic substances are weakly repelled by a magnet
  - Weak, so only present when ferromagnetism or paramagnetism do not exist

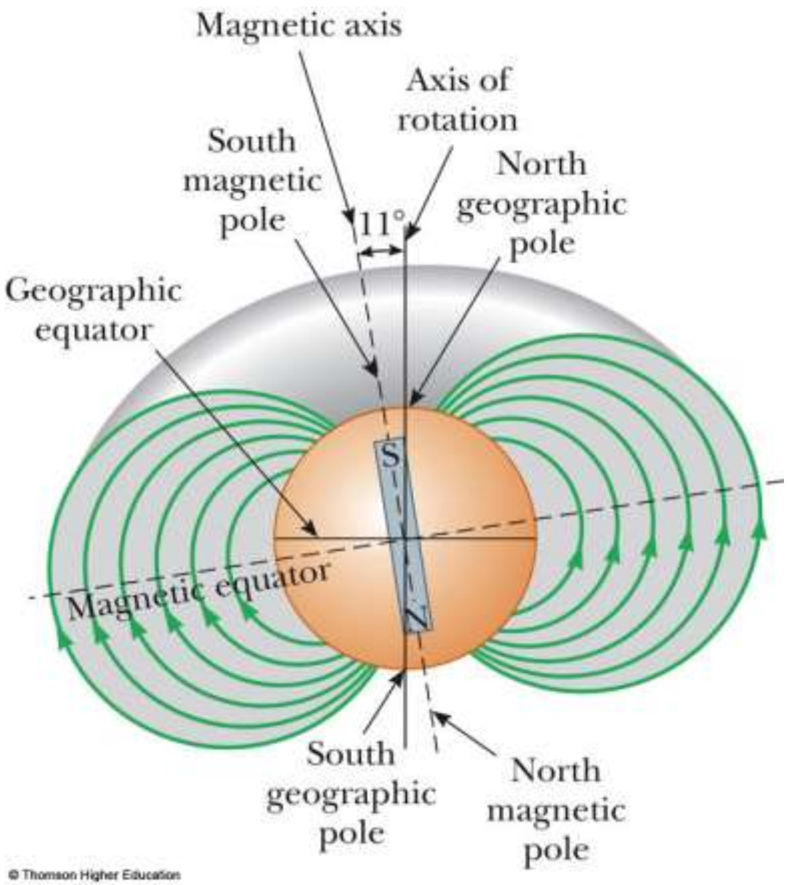
Meissner Effect

- Certain types of superconductors also exhibit perfect diamagnetism in the superconducting state
  - This is called the **Meissner effect**
- If a permanent magnet is brought near a superconductor, the two objects repel each other



Earth's Magnetic Field

- The Earth's magnetic field resembles that achieved by burying a huge bar magnet deep in the Earth's interior
- The Earth's south magnetic pole is located near the north geographic pole
- The Earth's north magnetic pole is located near the south geographic pole



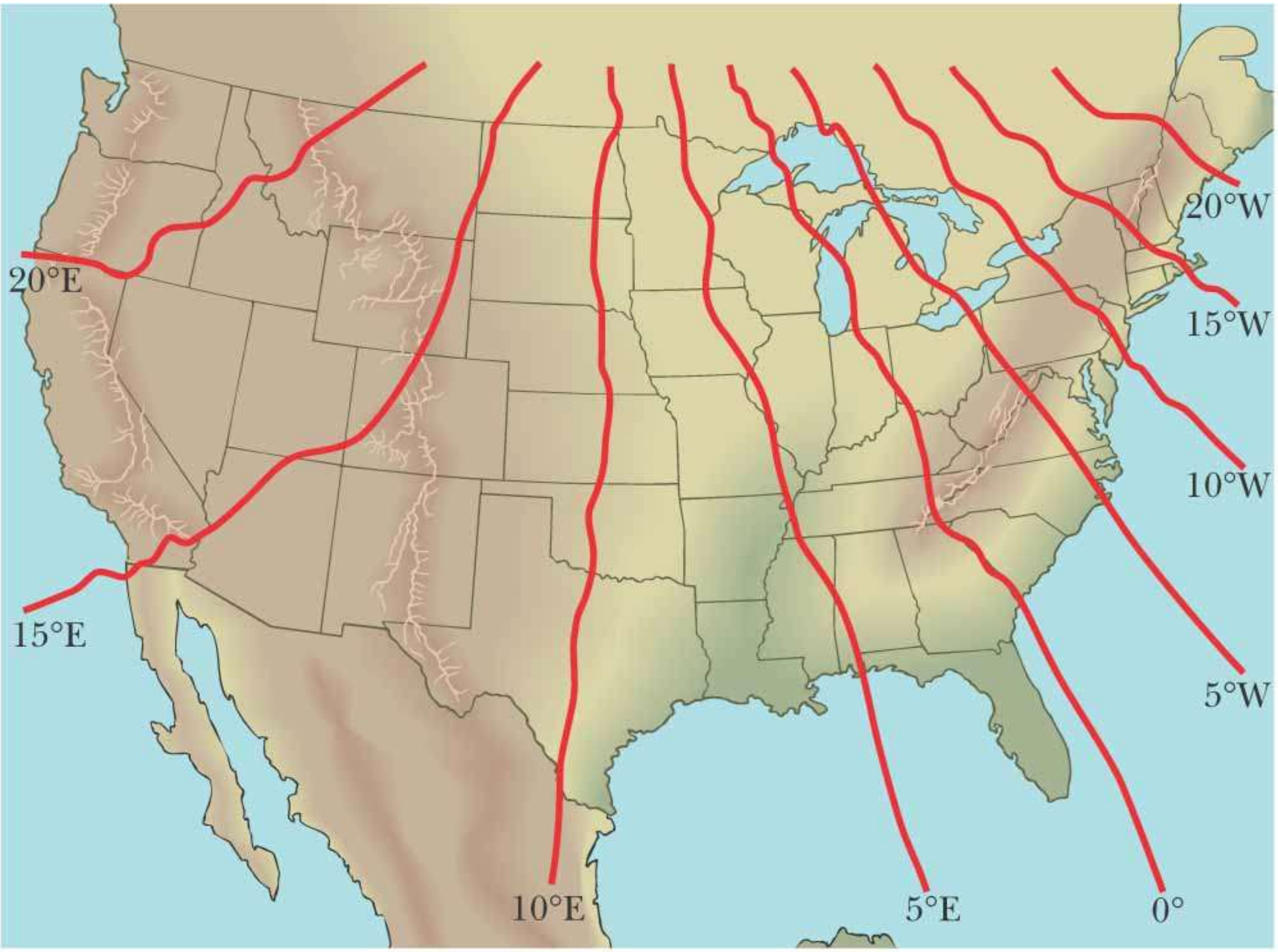
Vertical Movement of a Compass

- If a compass is free to rotate vertically as well as horizontally, it points to the Earth's surface
- The farther north the device is moved, the farther from horizontal the compass needle would be
  - The compass needle would be horizontal at the equator
  - The compass needle would point straight down at the magnetic pole

More About the Earth's Magnetic Poles

- The compass needle with point straight downward found at a point just north of Hudson Bay in Canada
  - This is considered to be the location of the south magnetic pole
  - The exact location varies slowly with time
- The magnetic and geographic poles are not in the same exact location
  - The difference between true north, at the geographic north pole, and magnetic north is called the magnetic declination
    - The amount of declination varies by location on the Earth's surface

Earth's Magnetic Declination



Source of the Earth's Magnetic Field

- There cannot be large masses of permanently magnetized materials since the high temperatures of the core prevent materials from retaining permanent magnetization
- The most likely source of the Earth's magnetic field is believed to be convection currents in the liquid part of the core
- There is also evidence that the planet's magnetic field is related to its rate of rotation

Reversals of the Earth's Magnetic Field

- The direction of the Earth's magnetic field reverses every few million years
  - Evidence of these reversals are found in basalts resulting from volcanic activity
  - The rocks provide a timeline for the periodic reversals of the field
    - The rocks are dated by other means to determine the timeline