Final Examination Paper

Department	Basic & Applied Science	Date	02/06/2012
Lecturer	Mathematics Group	Marks	40
Course Title	Mathematics 1	Time Allowed	2 hours
Course Code	BA123	Start Time	09:00-11:00

Find $\dfrac{dy}{dx}$ **for each of the following functions (From Q1 to Q3):**

Q1 : $\qquad y = x^2 \; cosec^{-1}\sqrt{x} \;\; - \;\; sin^4 x \;\; .$

3
Marks

Q2 : $\qquad y = \sqrt[5]{\left(\dfrac{x^x \; tanh^2 x}{cosh^{-1}x \; (2-3x)} \right)} \;\; .$

3
Marks

Q3 : $x^3 - 4\,y^2\,x^5 + 5\,y^4 = 12$.

3
Marks

Q4 :   **If** $x = \cos\left(\dfrac{t}{1+t}\right)$ **and** $y = \sin\left(\dfrac{t}{1+t}\right)$, **Show that**

$$y'' = \frac{-1}{y^3} \quad .$$

3
Marks

Evaluate the following limits (From Q5 to Q6):

Q5 : $\quad \lim_{x \to 0} (\cos x)^{1/x^2}$.

Q6 : $\lim\limits_{x \to 1} \ (1-x) \ \tan\left(\dfrac{\pi \, x}{2}\right)$.

2
Marks

Q7 : **Find the n$^{\text{th}}$ derivative for** $y = \ln(2+4x)$.

4
Marks

Q8 :   **Using Maclaurin's expansion, Show that**

$$e^x \cos x = 1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4 + \cdots \quad .$$

4
Marks

Q9 :   **If** $z = ln\left(x^2 + y^2\right)$, **show that** $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$.
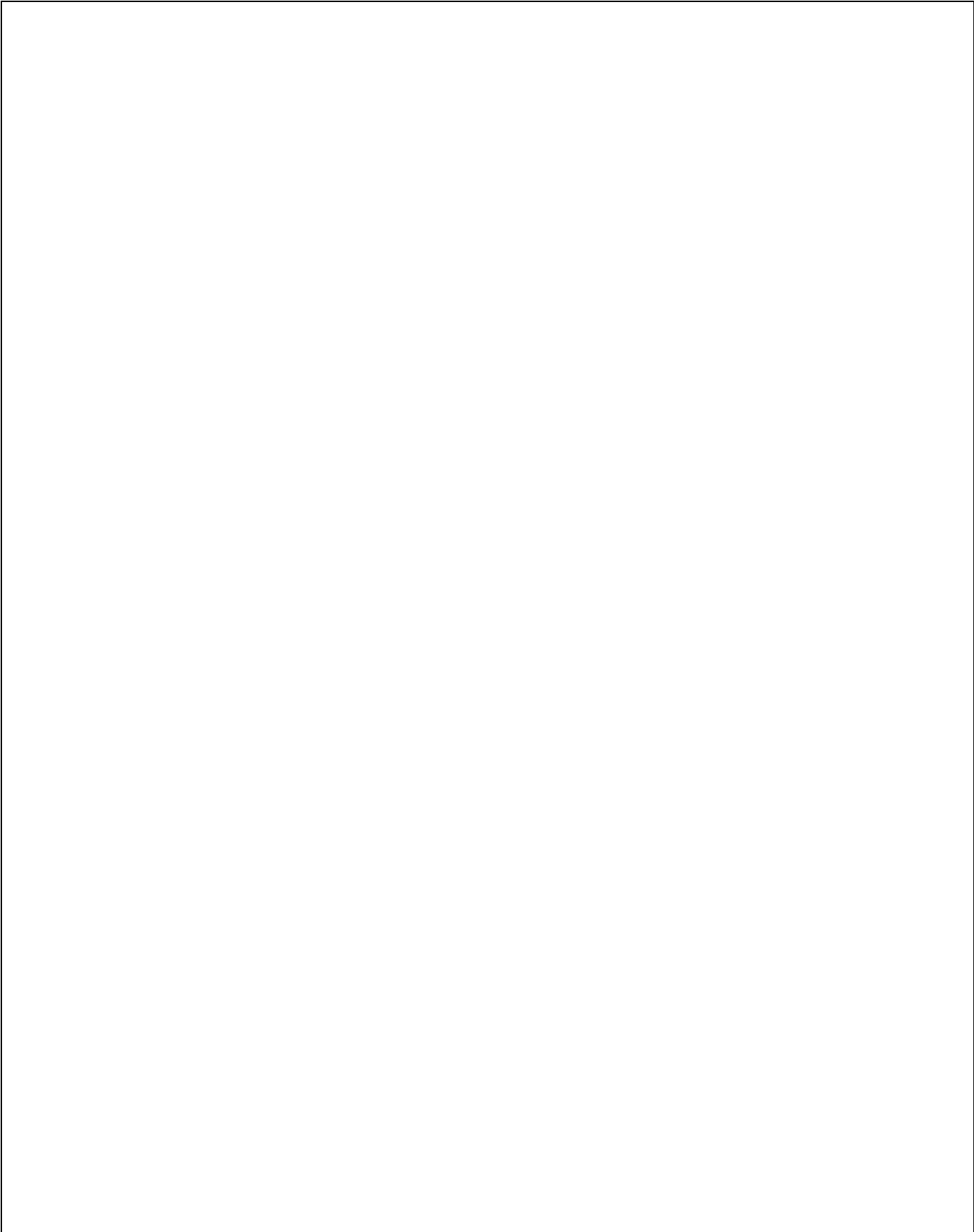
5
Marks

Q10 : **For the curve** $y = x^3 + 3x^2 + 4$

(a) Find the critical points.
(b) Find the intervals in which the curve is increasing and decreasing.
(c) Find the local maximum and minimum points.
(d) Find the inflection point.
(e) Find the concavity of the curve.
(f) Sketch the curve.

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Q11 :   **Discuss and sketch the curve** $x^2 + 2x - 4y - 7 = 0$.
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