Chapter 37

Interference of Light Waves

Wave Optics

- Wave optics is a study concerned with phenomena that cannot be adequately explained by geometric (ray) optics
- These phenomena include:
  - Interference
  - Diffraction
  - Polarization

Interference

- In *constructive interference* the amplitude of the resultant wave is greater than that of either individual wave
- In *destructive interference* the amplitude of the resultant wave is less than that of either individual wave
- All interference associated with light waves arises when the electromagnetic fields that constitute the individual waves combine

Conditions for Interference

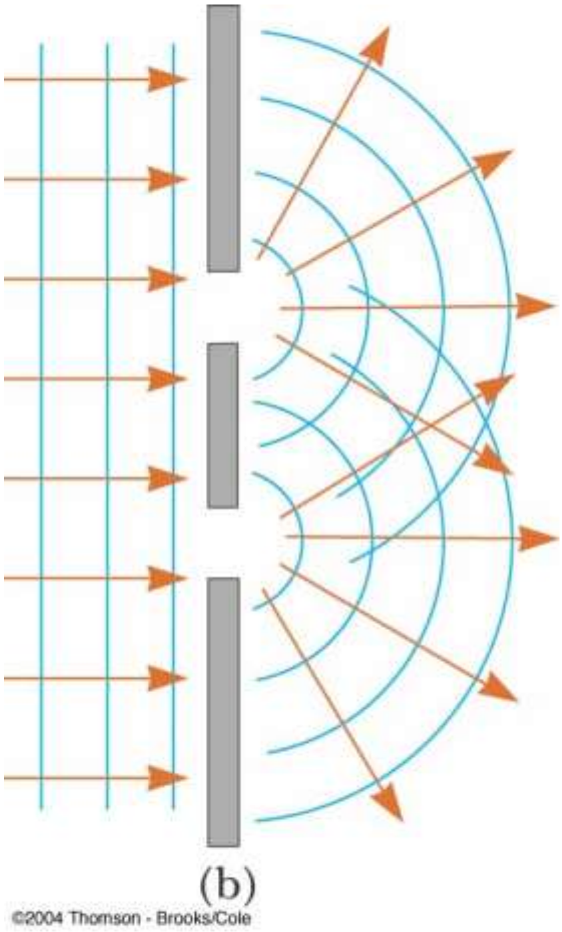
- To observe interference in light waves, the following two conditions must be met:
  - 1) The sources must be **coherent**
    - They must maintain a constant phase with respect to each other
  - 2) The sources should be **monochromatic**
    - Monochromatic means they have a single wavelength

Producing Coherent Sources

- Light from a monochromatic source is used to illuminate a barrier
- The barrier contains two narrow slits
  - The slits are small openings
- The light emerging from the two slits is coherent since a single source produces the original light beam
- This is a commonly used method

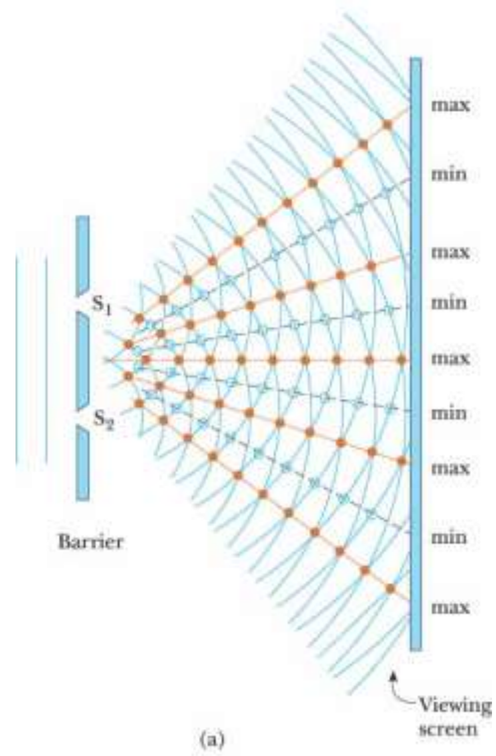
Diffraction

- From Huygens's principle we know the waves spread out from the slits
- This divergence of light from its initial line of travel is called **diffraction**



Young's Double-Slit Experiment: Schematic

- Thomas Young first demonstrated interference in light waves from two sources in 1801
- The narrow slits S_1 and S_2 act as sources of waves
- The waves emerging from the slits originate from the same wave front and therefore are always in phase



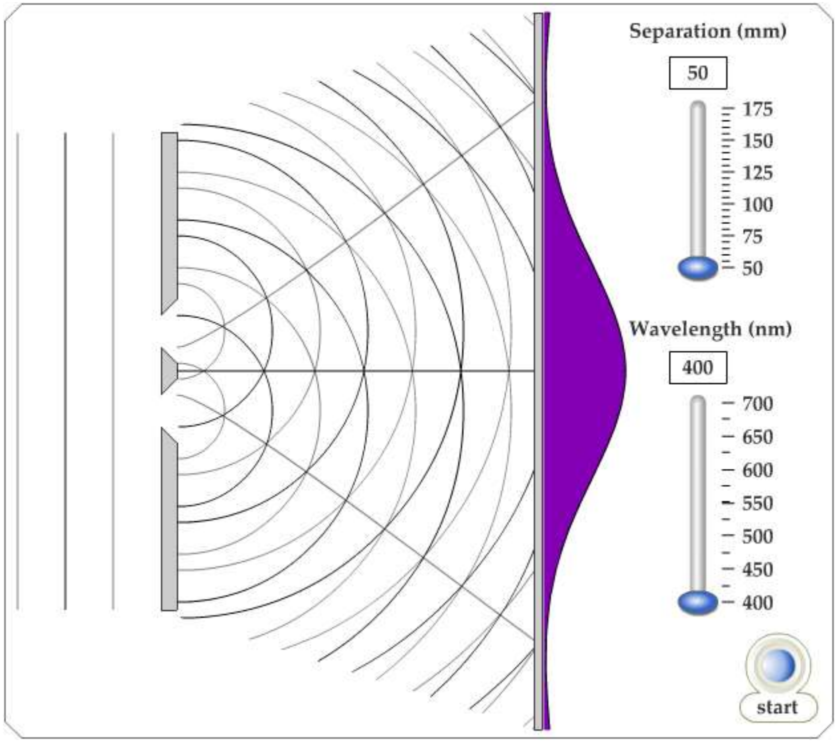
Resulting Interference Pattern

- The light from the two slits forms a visible pattern on a screen
- The pattern consists of a series of bright and dark parallel bands called *fringes*
- *Constructive interference* occurs where a bright fringe occurs
- *Destructive interference* results in a dark fringe

PLAY ACTIVE FIGURE

Active Figure 37.2

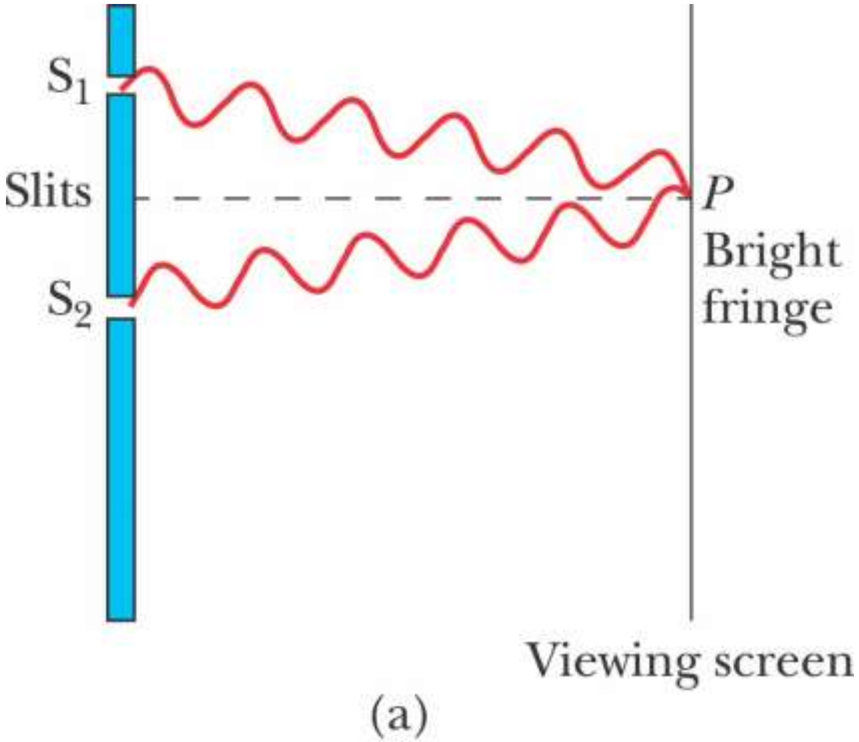
- Use the active figure to vary slit separation and the wavelength
- Observe the effect on the interference pattern



PLAY
ACTIVE FIGURE

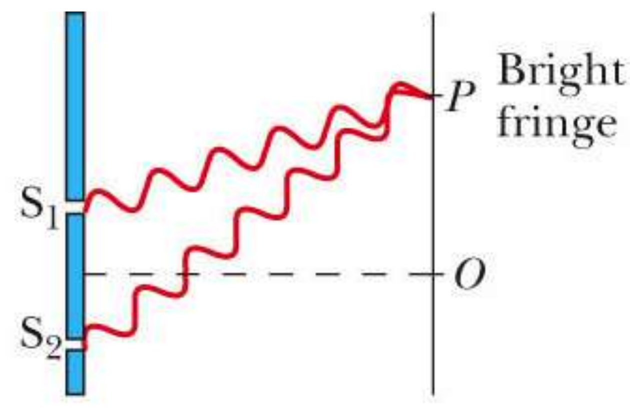
Interference Patterns

- Constructive interference occurs at point P
- The two waves travel the same distance
  - Therefore, they arrive in phase
- As a result, constructive interference occurs at this point and a bright fringe is observed



Interference Patterns, 2

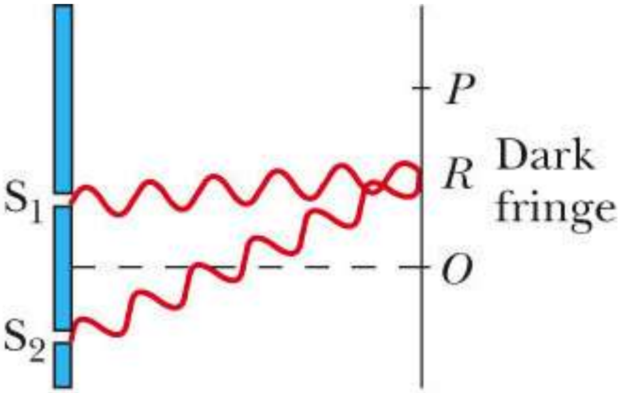
- The lower wave has to travel farther than the upper wave to reach point *P*
- The lower wave travels one wavelength farther
  - Therefore, the waves arrive in phase
- A second bright fringe occurs at this position



(b)

Interference Patterns, 3

- The upper wave travels one-half of a wavelength farther than the lower wave to reach point *R*
- The trough of the upper wave overlaps the crest of the lower wave
- This is destructive interference
  - A dark fringe occurs



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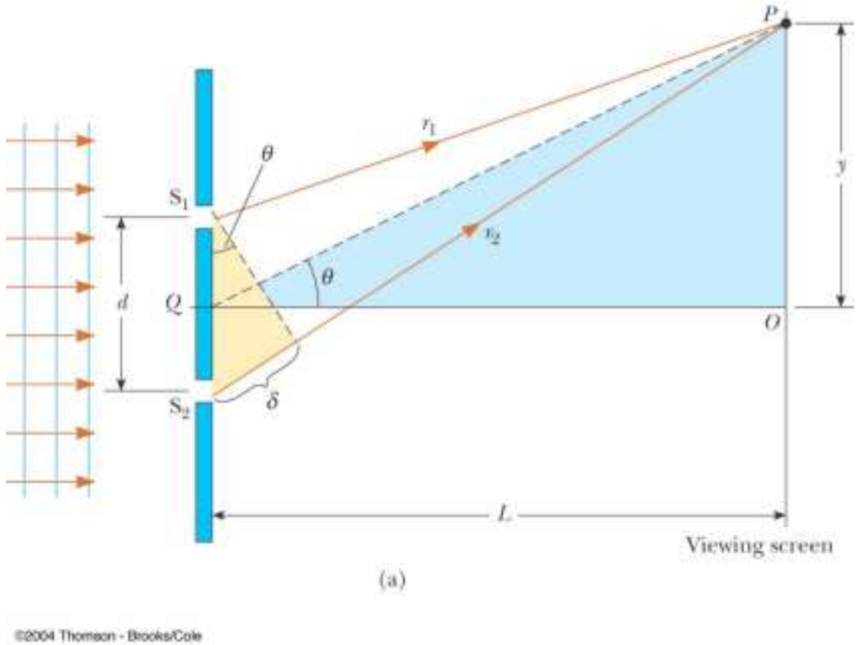
Young's Double-Slit Experiment: Geometry

- The path difference, δ, is found from the tan triangle
- $\delta = r_2 - r_1 = d \sin\theta$
  - This assumes the paths are parallel
  - Not exactly true, but a very good approximation if *L* is much greater than *d*



Interference Equations

- For a bright fringe produced by constructive interference, the path difference must be either zero or some integral multiple of the wavelength
- $\delta = d \sin \theta_{bright} = m\lambda$
  - m = 0, ±1, ±2, …
  - m is called the order number
    - When m = 0, it is the zeroth-order maximum
    - When m = ±1, it is called the first-order maximum

Interference Equations, 2

- When destructive interference occurs, a dark fringe is observed
- This needs a path difference of an odd half wavelength
- $\delta = d \sin \theta_{dark} = (m + \frac{1}{2})\lambda$
  - m = 0, ±1, ±2, …

Interference Equations, 4

- The positions of the fringes can be measured vertically from the zeroth-order maximum
- Using the blue triangle
  - $y_{bright} = L \tan \theta_{bright}$
  - $y_{dark} = L \tan \theta_{dark}$

Interference Equations, final

- Assumptions in a Young's Double Slit Experiment
  - L >> d
  - d >> λ
- Approximation:
  - θ is small and therefore the small angle approximation tan θ ~ sin θ can be used
- y = L tan θ ≈ L sin θ
- For bright fringes $y_{\text{bright}} = \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2 \ldots)$

Uses for Young's Double-Slit Experiment

- Young's double-slit experiment provides a method for measuring wavelength of the light
- This experiment gave the wave model of light a great deal of credibility
  - It was inconceivable that particles of light could cancel each other in a way that would explain the dark fringes

Intensity Distribution: Double-Slit Interference Pattern

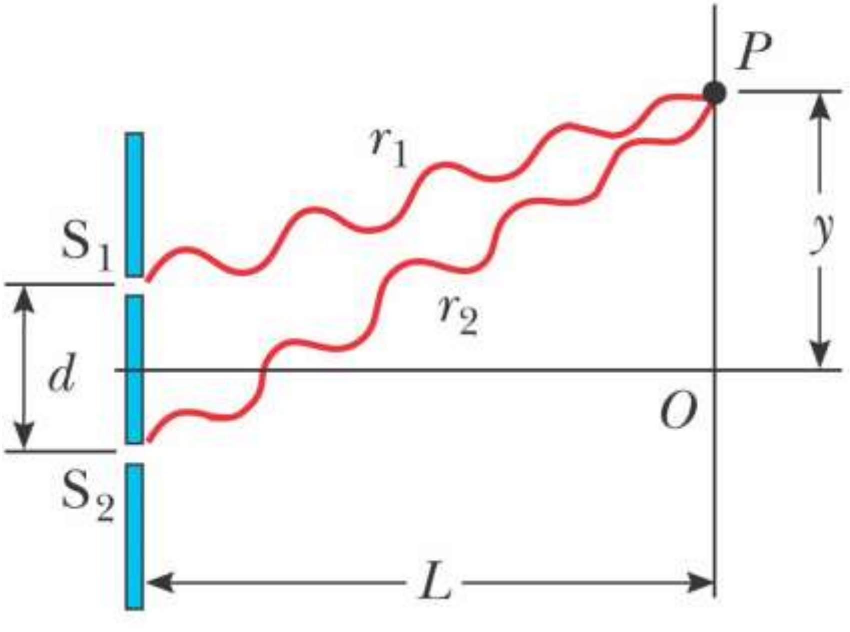
- The bright fringes in the interference pattern do not have sharp edges
  - The equations developed give the location of only the centers of the bright and dark fringes
- We can calculate the distribution of light intensity associated with the double-slit interference pattern

Intensity Distribution, Assumptions

- Assumptions:
  - The two slits represent coherent sources of sinusoidal waves
  - The waves from the slits have the same angular frequency, ω
  - The waves have a constant phase difference, ϕ
- The total magnitude of the electric field at any point on the screen is the superposition of the two waves

Intensity Distribution, Electric Fields

- The magnitude of each wave at point *P* can be found
  - $E_1 = E_o \sin \omega t$
  - $E_2 = E_o \sin (\omega t + \varphi)$
  - Both waves have the same amplitude, E_o



Intensity Distribution, Phase Relationships

- The phase difference between the two waves at P depends on their path difference
  - $\delta = r_2 - r_1 = d \sin \theta$
- A path difference of λ (for constructive interference) corresponds to a phase difference of 2π rad
- A path difference of δ is the same fraction of λ as the phase difference φ is of 2π
- This gives $\varphi = \dfrac{2\pi}{\lambda}\delta = \dfrac{2\pi}{\lambda} d \sin \theta$

Intensity Distribution, Resultant Field

- The magnitude of the resultant electric field comes from the superposition principle
  - $E_P = E_1 + E_2 = E_o[\sin \omega t + \sin(\omega t + \varphi)]$
- This can also be expressed as

$$E_P = 2E_o \cos\left(\frac{\varphi}{2}\right)\sin\left(\omega t + \frac{\varphi}{2}\right)$$

  - E_P has the same frequency as the light at the slits
  - The magnitude of the field is multiplied by the factor 2 cos (φ / 2)

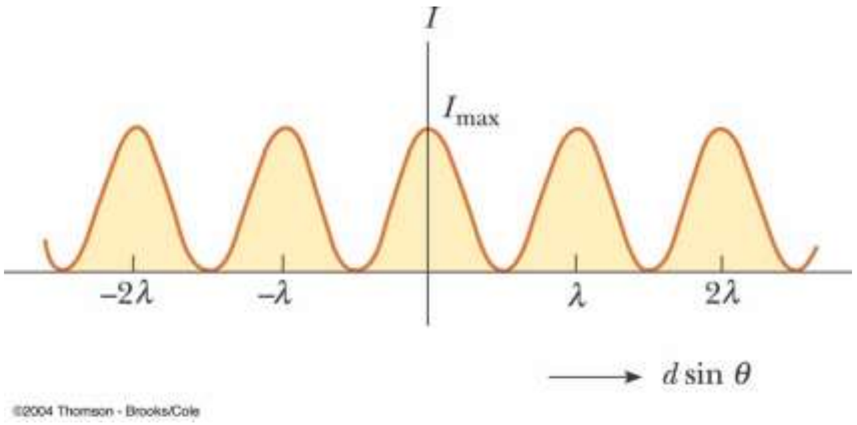
Intensity Distribution, Equation

- The expression for the intensity comes from the fact that the *intensity of a wave is proportional to the square of the resultant electric field magnitude at that point*
- The intensity therefore is

$$I = I_{max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) \approx I_{max} \cos^2\left(\frac{\pi d}{\lambda L} y\right)$$

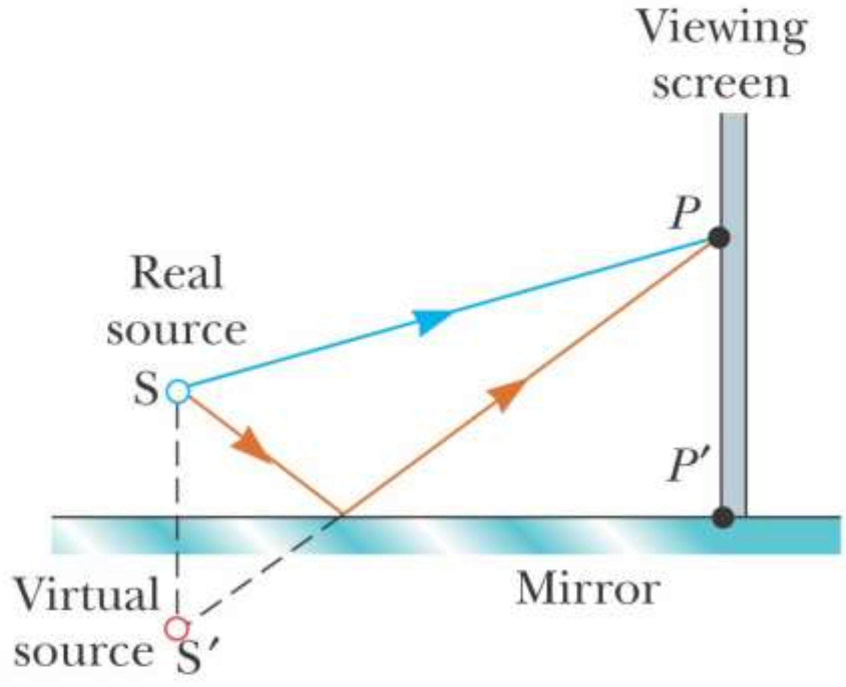
Light Intensity, Graph

- The interference pattern consists of equally spaced fringes of equal intensity
- This result is valid only if *L* >> *d* and for small values of θ



Lloyd's Mirror

- An arrangement for producing an interference pattern with a single light source
- Waves reach point *P* either by a direct path or by reflection
- The reflected ray can be treated as a ray from the source S' behind the mirror

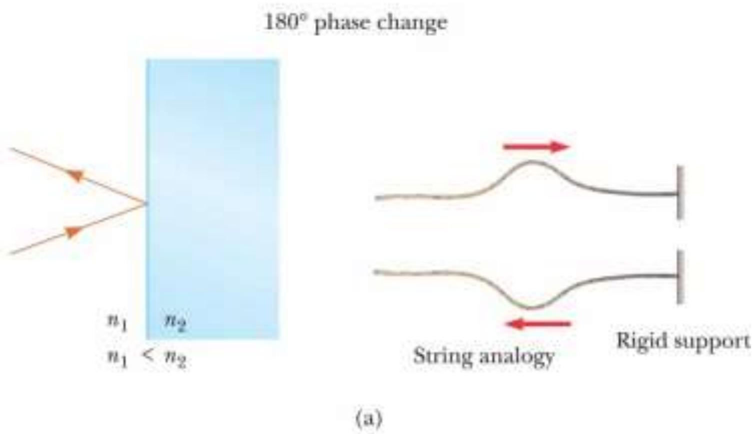


Interference Pattern from a Lloyd's Mirror

- This arrangement can be thought of as a double-slit source with the distance between points S and S' comparable to length d
- An interference pattern is formed
- The positions of the dark and bright fringes are reversed relative to the pattern of two real sources
- This is because there is a 180° phase change produced by the reflection

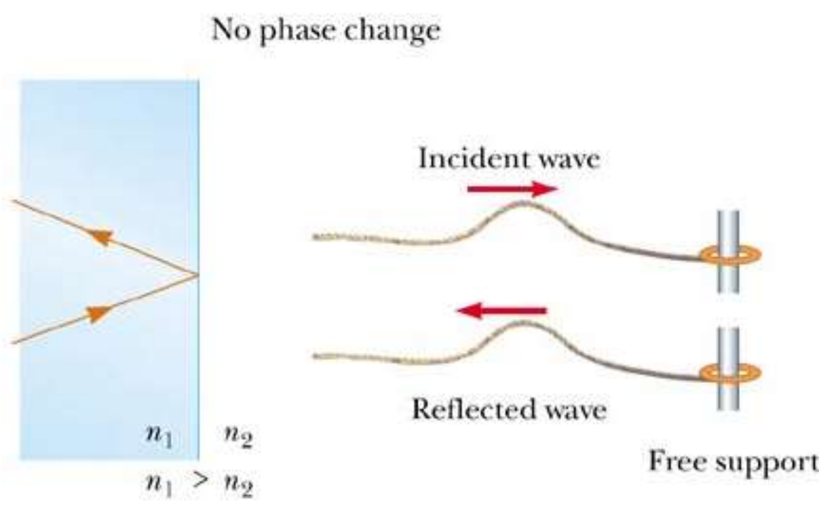
Phase Changes Due To Reflection

- An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling
  - Analogous to a pulse on a string reflected from a rigid support



Phase Changes Due To Reflection, cont.

- There is no phase change when the wave is reflected from a boundary leading to a medium of lower index of refraction
  - Analogous to a pulse on a string reflecting from a free support



Interference in Thin Films

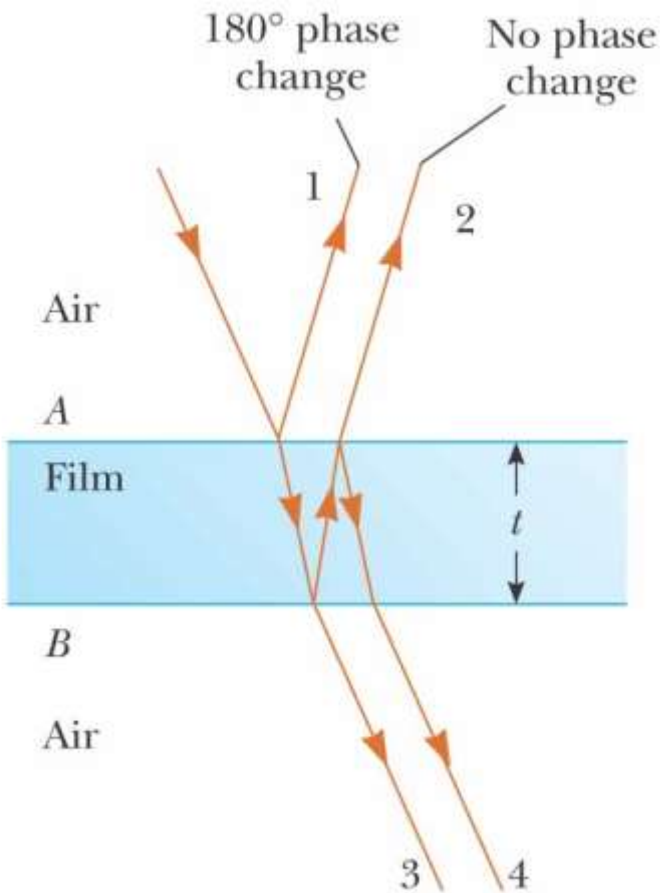
- Interference effects are commonly observed in thin films
  - Examples include soap bubbles and oil on water
- The various colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film

Interference in Thin Films, 2

- Facts to remember
  - An electromagnetic wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change on reflection when $n_2 > n_1$
    - There is no phase change in the reflected wave if $n_2 < n_1$
  - The wavelength of light λ_n in a medium with index of refraction n is $\lambda_n = \lambda/n$ where λ is the wavelength of light in vacuum

Interference in Thin Films, 3

- Assume the light rays are traveling in air nearly normal to the two surfaces of the film
- Ray 1 undergoes a phase change of 180° with respect to the incident ray
- Ray 2, which is reflected from the lower surface, undergoes no phase change with respect to the incident wave



Interference in Thin Films, 4

- Ray 2 also travels an additional distance of 2t before the waves recombine
- For constructive interference
  - $2nt = (m + \frac{1}{2})\lambda$ $(m = 0, 1, 2 \ldots)$
    - This takes into account both the difference in optical path length for the two rays and the 180° phase change
- For destructive interference
  - $2nt = m\lambda$ $(m = 0, 1, 2 \ldots)$

Interference in Thin Films, 5

- Two factors influence interference
  - Possible phase reversals on reflection
  - Differences in travel distance
- The conditions are valid if the medium above the top surface is the same as the medium below the bottom surface
  - If there are different media, these conditions are valid as long as the index of refraction for both is less than n

Interference in Thin Films, 6

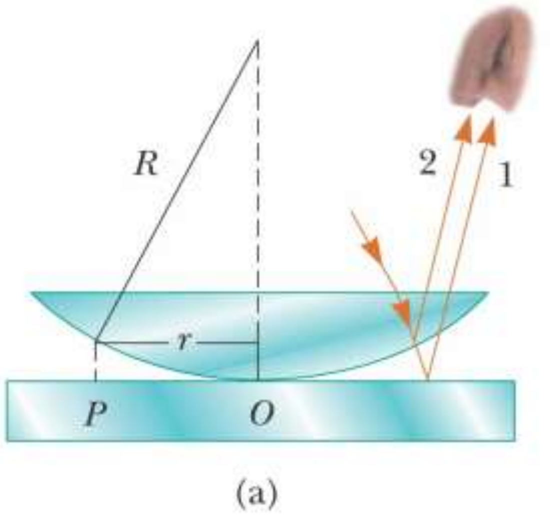
- If the thin film is between two different media, one of lower index than the film and one of higher index, the conditions for constructive and destructive interference are reversed
- With different materials on either side of the film, you may have a situation in which there is a 180^{o} phase change at both surfaces or at neither surface
  - Be sure to check both the path length and the phase change

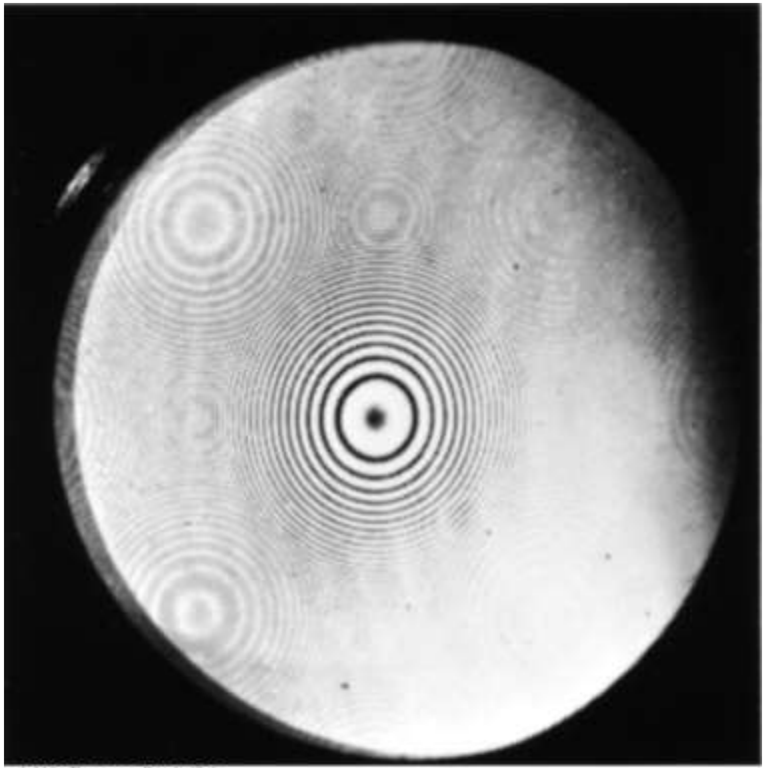
Interference in Thin Film, Soap Bubble Example

Newton’s Rings

- Another method for viewing interference is to place a plano-convex lens on top of a flat glass surface
- The air film between the glass surfaces varies in thickness from zero at the point of contact to some thickness t
- A pattern of light and dark rings is observed
  - These rings are called Newton’s rings
  - The particle model of light could not explain the origin of the rings
- Newton’s rings can be used to test optical lenses

Newton's Rings, Set-Up and Pattern





Problem Solving Strategy with Thin Films, 1

- *Conceptualize*
  - Identify the light source
  - Identify the location of the observer
- *Categorize*
  - Be sure the techniques for thin-film interference are appropriate
  - Identify the thin film causing the interference

Problem Solving with Thin Films, 2

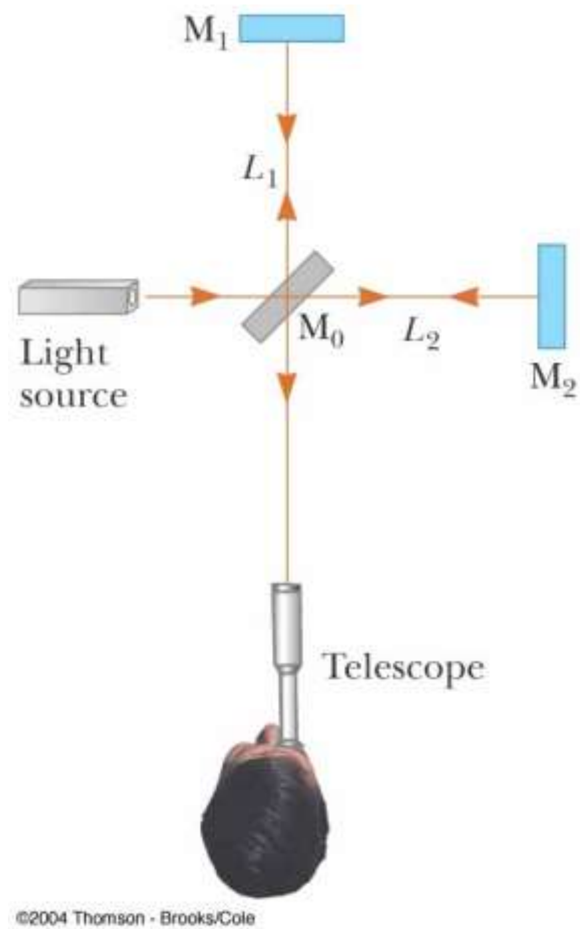
- *Analyze*
  - The type of interference – constructive or destructive – that occurs is determined by the phase relationship between the upper and lower surfaces
  - Phase differences have two causes
    - differences in the distances traveled
    - phase changes occurring on reflection
  - Both causes must be considered when determining constructive or destructive interference
  - Use the indices of refraction of the materials to determine the correct equations
- *Finalize*
  - Be sure your results make sense physically
  - Be sure they are of an appropriate size

Michelson Interferometer

- The interferometer was invented by an American physicist, A. A. Michelson
- The interferometer splits light into two parts and then recombines the parts to form an interference pattern
- The device can be used to measure wavelengths or other lengths with great precision

Michelson Interferometer, Schematic

- A ray of light is split into two rays by the mirror M_o
  - The mirror is at 45º to the incident beam
  - The mirror is called a *beam splitter*
- It transmits half the light and reflects the rest

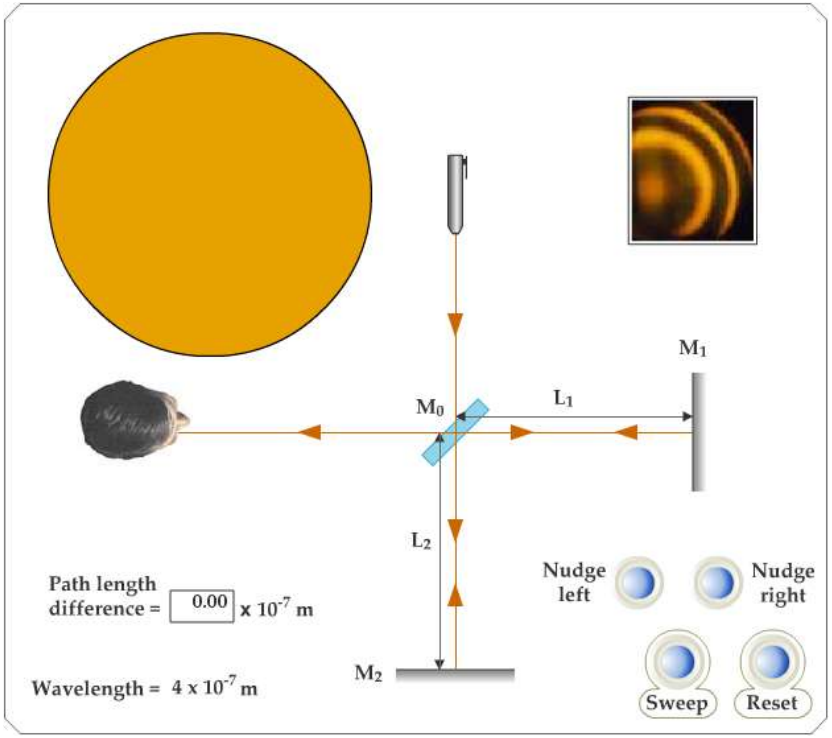


Michelson Interferometer, Schematic Explanation, cont.

- The reflected ray goes toward mirror M_1
- The transmitted ray goes toward mirror M_2
- The two rays travel separate paths L_1 and L_2
- After reflecting from M_1 and M_2, the rays eventually recombine at M_o and form an interference pattern

Active Figure 37.14

- Use the active figure to move the mirror
- Observe the effect on the interference pattern
- Use the interferometer to measure the wavelength of the light



PLAY ACTIVE FIGURE

Michelson Interferometer – Operation

- The interference condition for the two rays is determined by their path length difference
- M_1 is moveable
- As it moves, the fringe pattern collapses or expands, depending on the direction M_1 is moved

Michelson Interferometer – Operation, cont.

- The fringe pattern shifts by one-half fringe each time M_1 is moved a distance $\lambda/4$
- The wavelength of the light is then measured by counting the number of fringe shifts for a given displacement of M_1

Michelson Interferometer – Applications

- The Michelson interferometer was used to disprove the idea that the Earth moves through an ether
- Modern applications include
  - Fourier Transform Infrared Spectroscopy (FTIR)
  - Laser Interferometer Gravitational-Wave Observatory (LIGO)

Fourier Transform Infrared Spectroscopy

- This is used to create a high-resolution spectrum in a very short time interval
- The result is a complex set of data relating light intensity as a function of mirror position
  - This is called an interferogram
- The interferogram can be analyzed by a computer to provide all of the wavelength components
  - This process is called a Fourier transform

Laser Interferometer Gravitational-Wave Observatory

- General relativity predicts the existence of gravitational waves
- In Einstein's theory, gravity is equivalent to a distortion of space
  - These distortions can then propagate through space
- The LIGO apparatus is designed to detect the distortion produced by a disturbance that passes near the Earth

LIGO, cont.

- The interferometer uses laser beams with an effective path length of several kilometers
- At the end of an arm of the interferometer, a mirror is mounted on a massive pendulum
- When a gravitational wave passes, the pendulum moves, and the interference pattern due to the laser beams from the two arms changes

LIGO in Richland, Washington