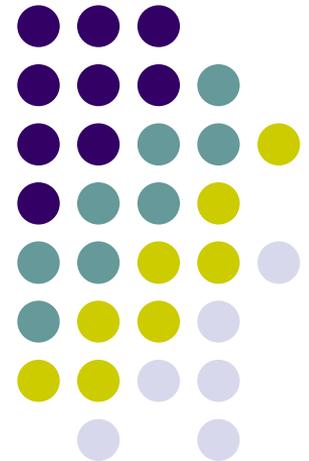


Chapter 33

Alternating Current Circuits





AC Circuits

- An AC circuit consists of a combination of circuit elements and a power source
- The power source provides an alternative voltage, Δv
- Notation Note
 - Lower case symbols will indicate instantaneous values
 - Capital letters will indicate fixed values



AC Voltage

- The output of an AC power source is sinusoidal and varies with time according to the following equation:
 - $\Delta v = \Delta V_{max} \sin \omega t$
 - Δv is the instantaneous voltage
 - ΔV_{max} is the maximum output voltage of the source
 - Also called the **voltage amplitude**
 - ω is the angular frequency of the AC voltage

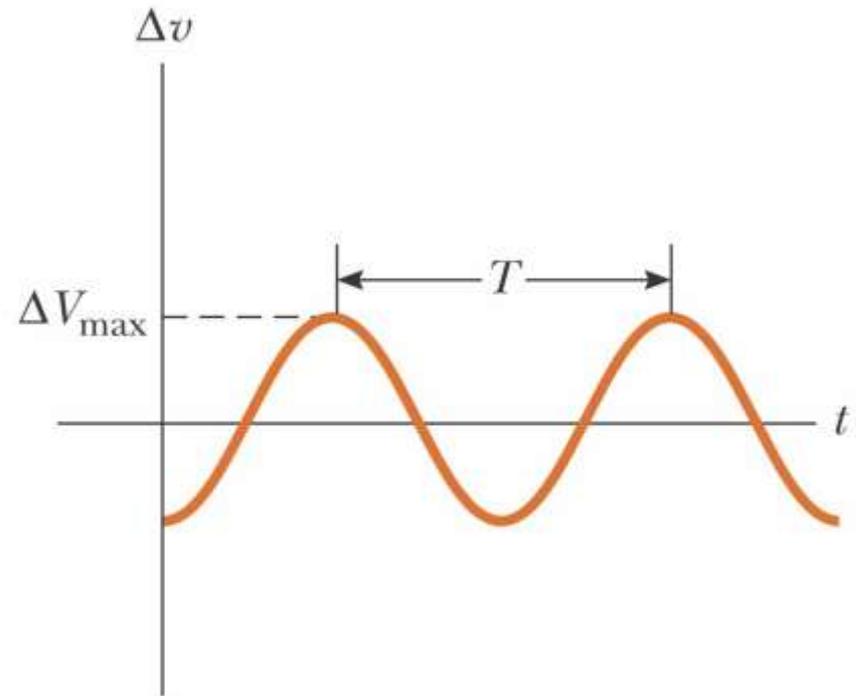
AC Voltage, cont.



- The angular frequency is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

- f is the frequency of the source
- T is the period of the source
- The voltage is positive during one half of the cycle and negative during the other half





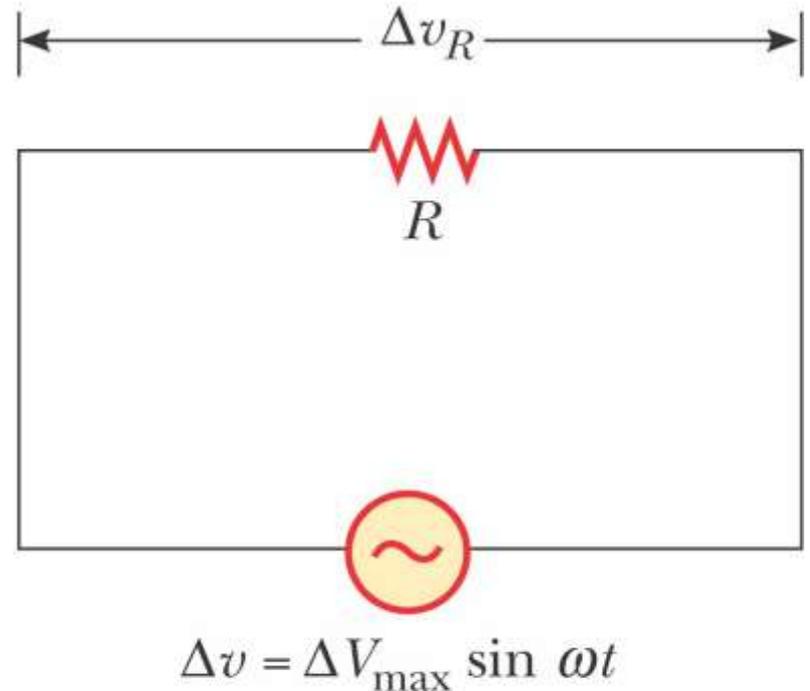
AC Voltage, final

- The current in any circuit driven by an AC source is an alternating current that varies sinusoidally with time
- Commercial electric power plants in the US use a frequency of 60 Hz
 - This corresponds with an angular frequency of 377 rad/s

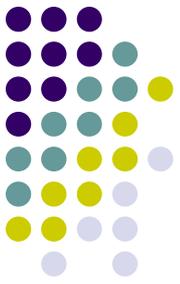


Resistors in an AC Circuit

- Consider a circuit consisting of an AC source and a resistor
- The AC source is symbolized by 
- $\Delta V_R = \Delta V_{max} = V_{max} \sin \omega t$
- ΔV_R is the instantaneous voltage across the resistor



Resistors in an AC Circuit, 2

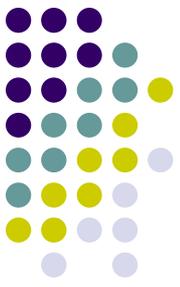


- The instantaneous current in the resistor is

$$i_R = \frac{\Delta V_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

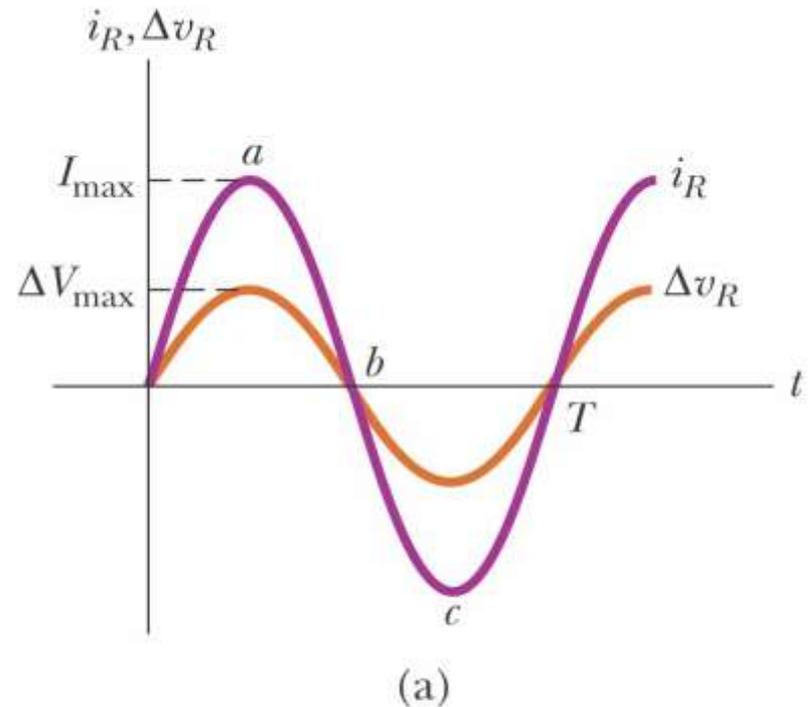
- The instantaneous voltage across the resistor is also given as

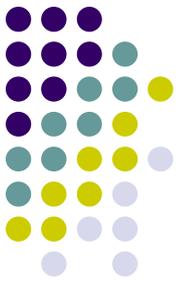
$$\Delta V_R = I_{max} R \sin \omega t$$



Resistors in an AC Circuit, 3

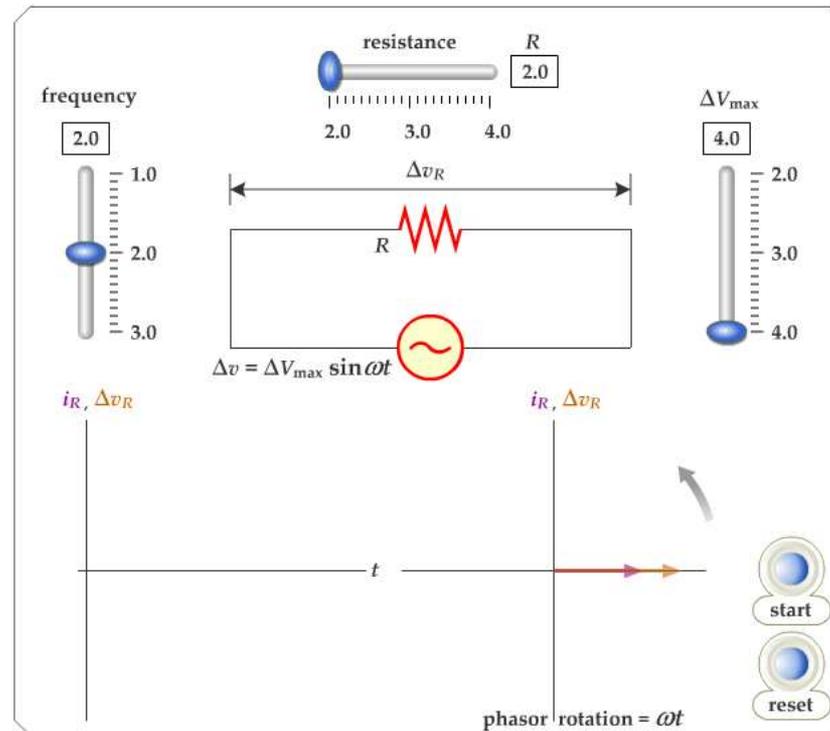
- The graph shows the current through and the voltage across the resistor
- The current and the voltage reach their maximum values at the same time
- The current and the voltage are said to be *in phase*





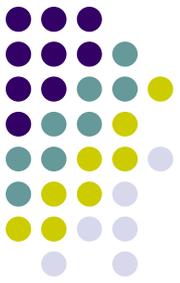
Active Figure 33.2

- Adjust the resistance, frequency and V_{\max}
- Observe the results with the graph and the phasor diagram

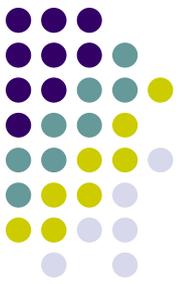


**PLAY
ACTIVE FIGURE**

Resistors in an AC Circuit, 4

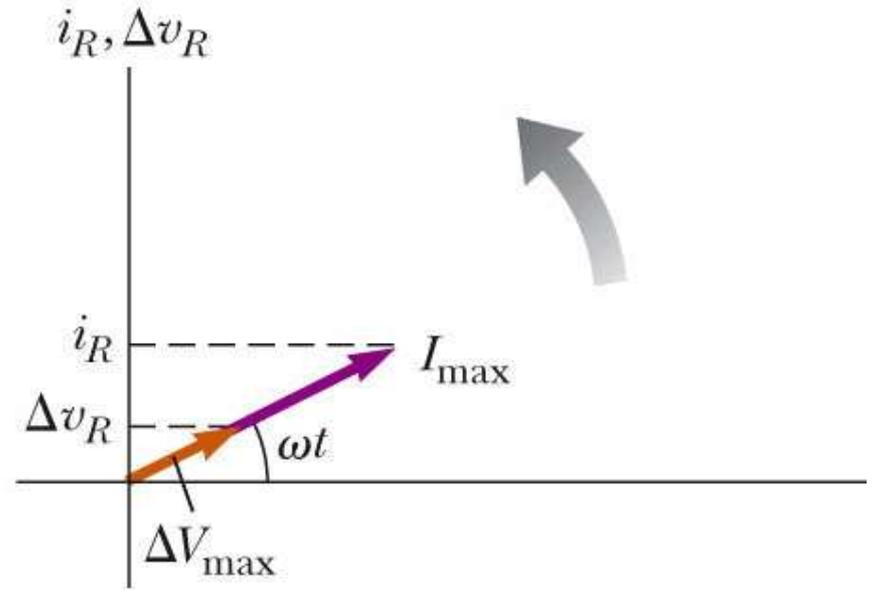


- For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor
- The direction of the current has no effect on the behavior of the resistor
- Resistors behave essentially the same way in both DC and AC circuits



Phasor Diagram

- To simplify the analysis of AC circuits, a graphical constructor called a *phasor diagram* can be used
- A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents

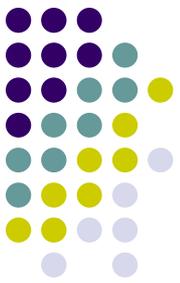


(b)



Phasors, cont.

- The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable
- The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents



rms Current and Voltage

- The average current in one cycle is zero
- The rms current is the average of importance in an AC circuit
 - rms stands for root mean square

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max}$$

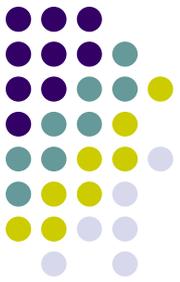
- Alternating voltages can also be discussed in terms of rms values

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707 \Delta V_{max}$$



Power

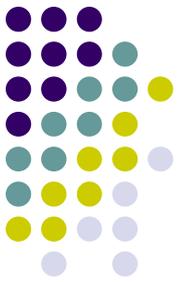
- The rate at which electrical energy is dissipated in the circuit is given by
 - $P = i^2 R$
 - i is the *instantaneous current*
 - The heating effect produced by an AC current with a maximum value of I_{\max} is not the same as that of a DC current of the same value
 - The maximum current occurs for a small amount of time



Power, cont.

- The average power delivered to a resistor that carries an alternating current is

$$P_{av} = I_{rms}^2 R$$



Notes About rms Values

- rms values are used when discussing alternating currents and voltages because
 - AC ammeters and voltmeters are designed to read rms values
 - Many of the equations that will be used have the same form as their DC counterparts



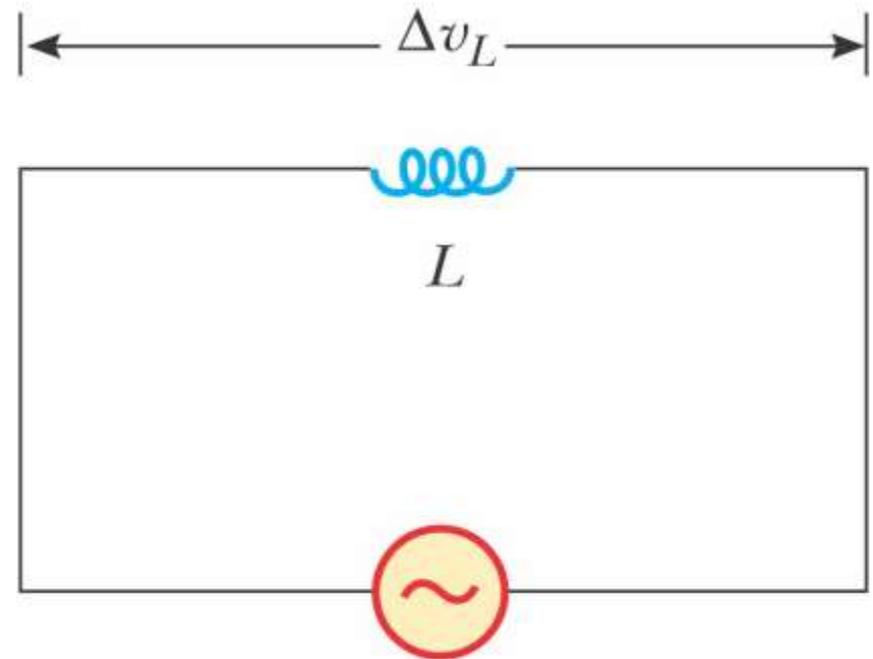
Inductors in an AC Circuit

- Kirchhoff's loop rule can be applied and gives:

$$\Delta v + \Delta v_L = 0, \text{ or}$$

$$\Delta v - L \frac{di}{dt} = 0$$

$$\Delta v = L \frac{di}{dt} = \Delta V_{max} \sin \omega t$$



$$\Delta v = \Delta V_{max} \sin \omega t$$



Current in an Inductor

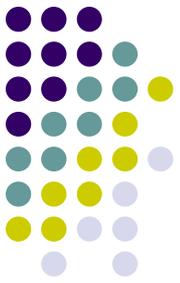
- The equation obtained from Kirchhoff's loop rule can be solved for the current

$$i_L = \frac{\Delta V_{\max}}{L} \int \sin \omega t \, dt = -\frac{\Delta V_{\max}}{\omega L} \cos \omega t$$

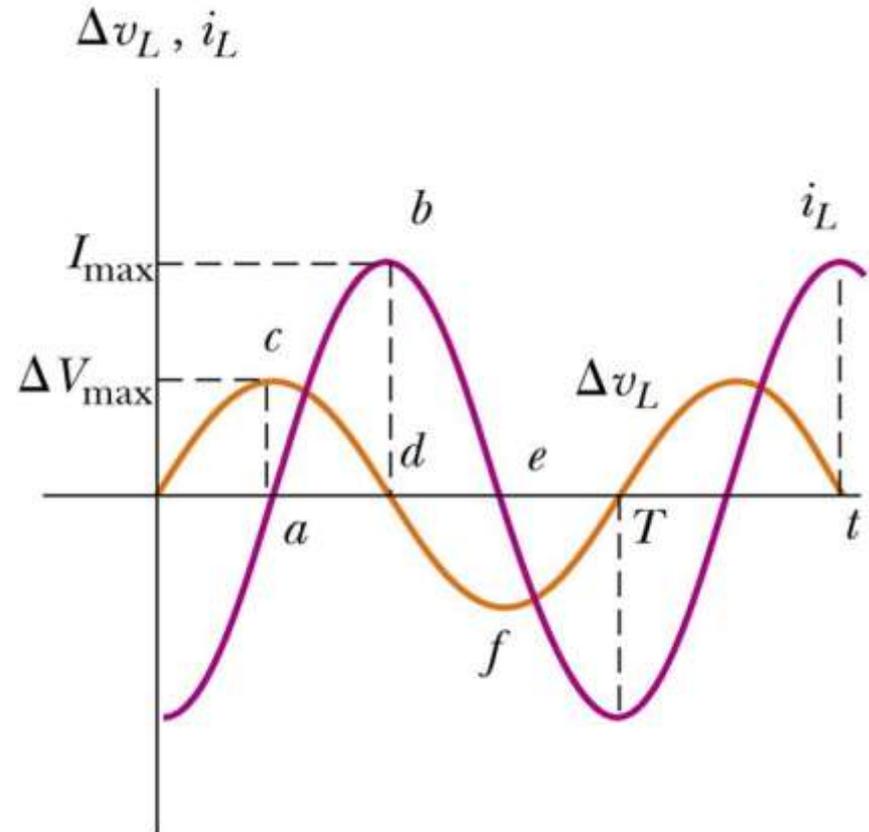
$$i_L = \frac{\Delta V_{\max}}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right) \quad I_{\max} = \frac{\Delta V_{\max}}{\omega L}$$

- This shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by $(\pi/2)$ rad = 90°

Phase Relationship of Inductors in an AC Circuit



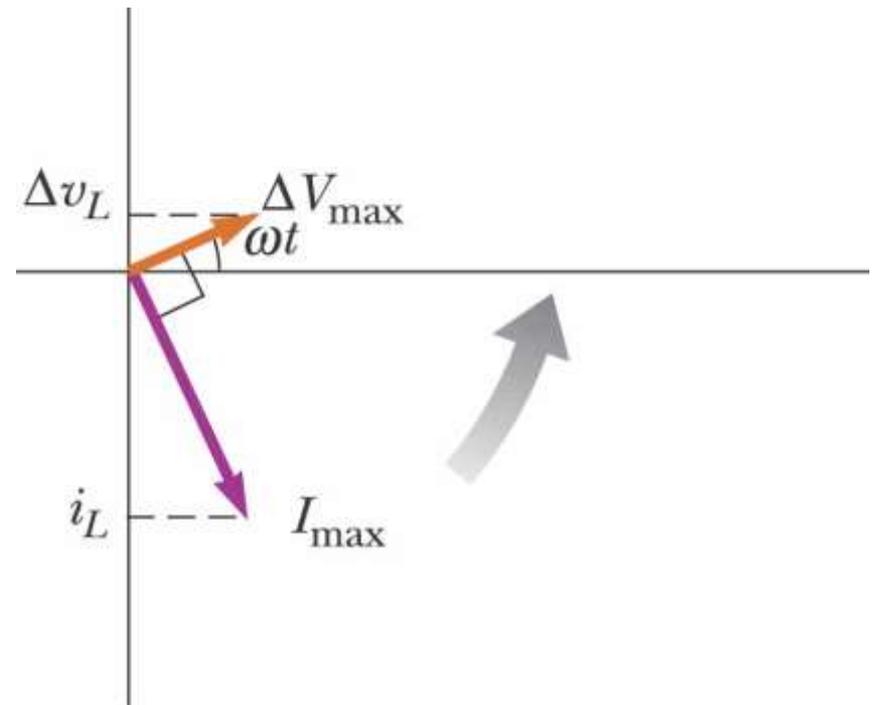
- The current is a maximum when the voltage across the inductor is zero
 - The current is momentarily not changing
- For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° ($\pi/2$)



Phasor Diagram for an Inductor



- The phasors are at 90° with respect to each other
- This represents the phase difference between the current and voltage
- Specifically, the current lags behind the voltage by 90°

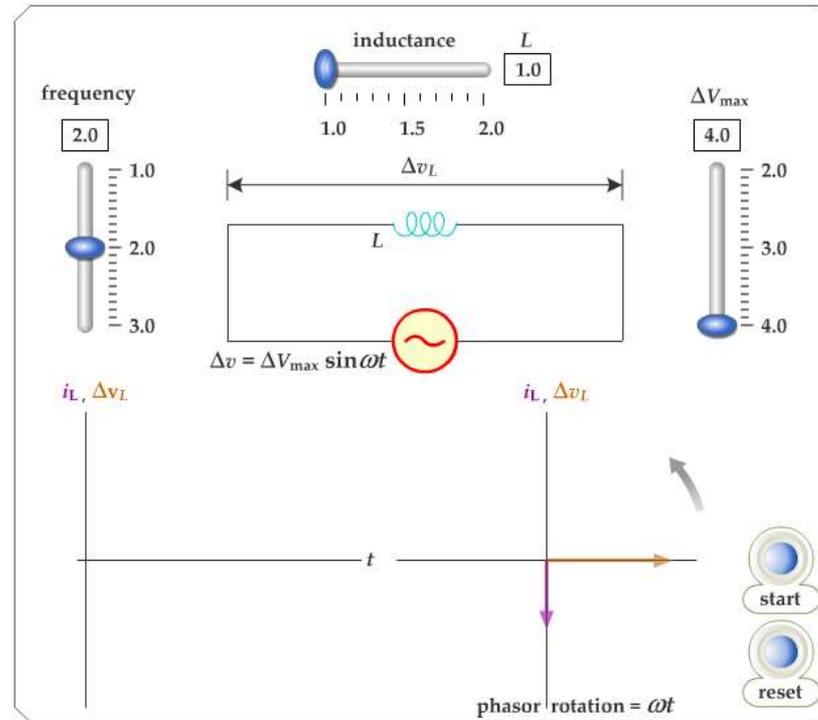


(b)



Active Figure 33.6

- Use the active figure to adjust the inductance, frequency and voltage
- Observe the results on the graph and phasor diagrams



**PLAY
ACTIVE FIGURE**



Inductive Reactance

- The factor ωL has the same units as resistance and is related to current and voltage in the same way as resistance
- Because ωL depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies
- The factor is the **inductive reactance** and is given by:
 - $X_L = \omega L$



Inductive Reactance, cont.

- Current can be expressed in terms of the inductive reactance

$$I_{\max} = \frac{\Delta V_{\max}}{X_L} \text{ or } I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L}$$

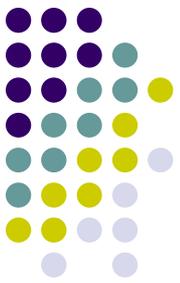
- As the frequency increases, the inductive reactance increases
 - This is consistent with Faraday's Law:
 - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the reactance and a decrease in the current



Voltage Across the Inductor

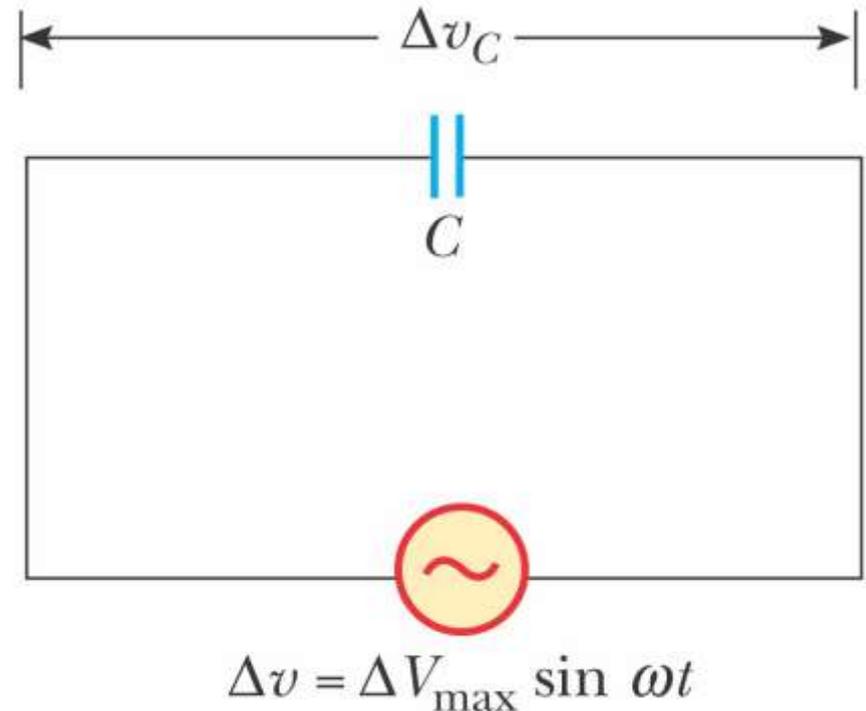
- The instantaneous voltage across the inductor is

$$\begin{aligned}\Delta v_L &= -L \frac{di}{dt} \\ &= -\Delta V_{\max} \sin \omega t \\ &= -I_{\max} X_L \sin \omega t\end{aligned}$$



Capacitors in an AC Circuit

- The circuit contains a capacitor and an AC source
- Kirchhoff's loop rule gives:
 $\Delta v + \Delta v_C = 0$ and so
 $\Delta v = \Delta v_C = \Delta V_{\max} \sin \omega t$
- Δv_C is the instantaneous voltage across the capacitor



Capacitors in an AC Circuit, cont.



- The charge is $q = C\Delta V_{max} \sin \omega t$
- The instantaneous current is given by

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{max} \cos \omega t$$

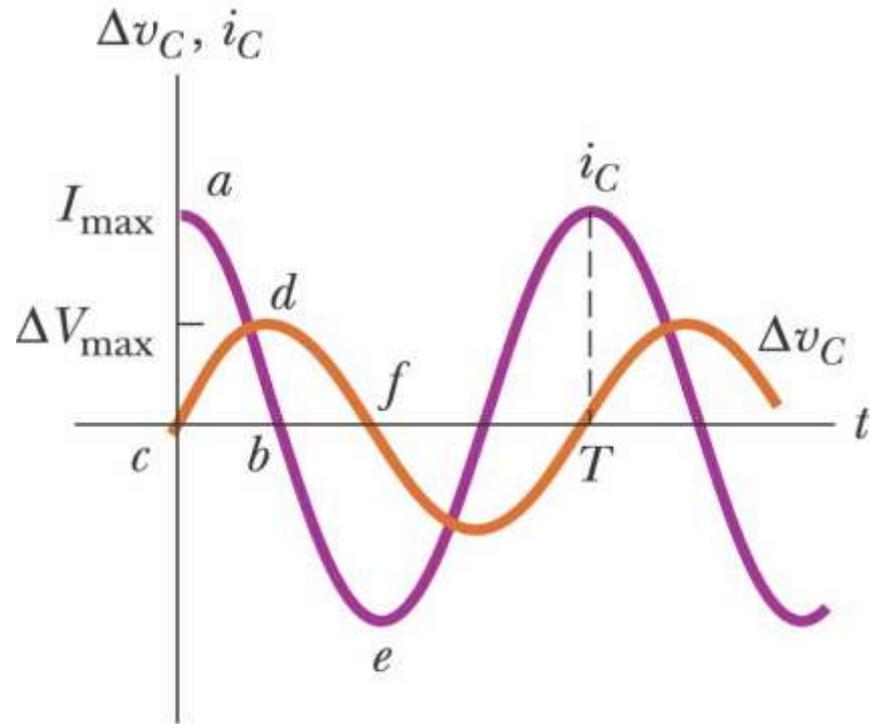
$$\text{or } i_C = \omega C \Delta V_{max} \sin \left(\omega t + \frac{\pi}{2} \right)$$

- The current is $\pi/2$ rad = 90° out of phase with the voltage

More About Capacitors in an AC Circuit



- The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value
- The current leads the voltage by 90°

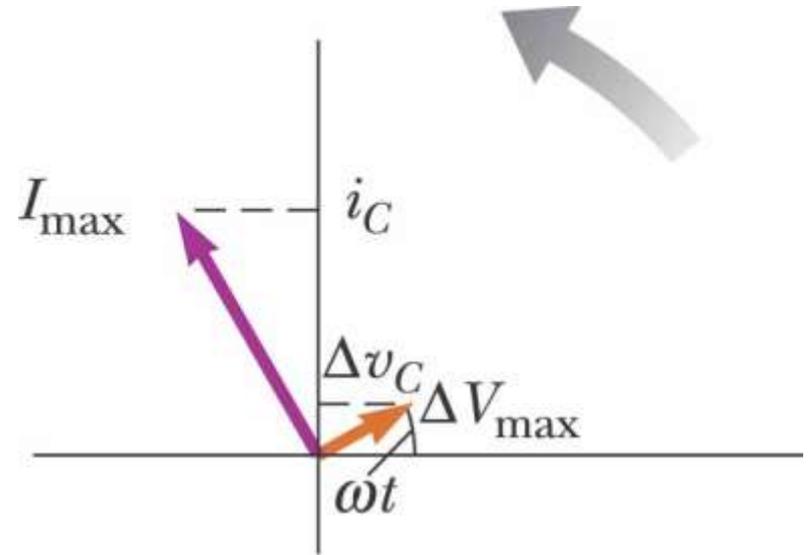


(a)



Phasor Diagram for Capacitor

- The phasor diagram shows that for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90°

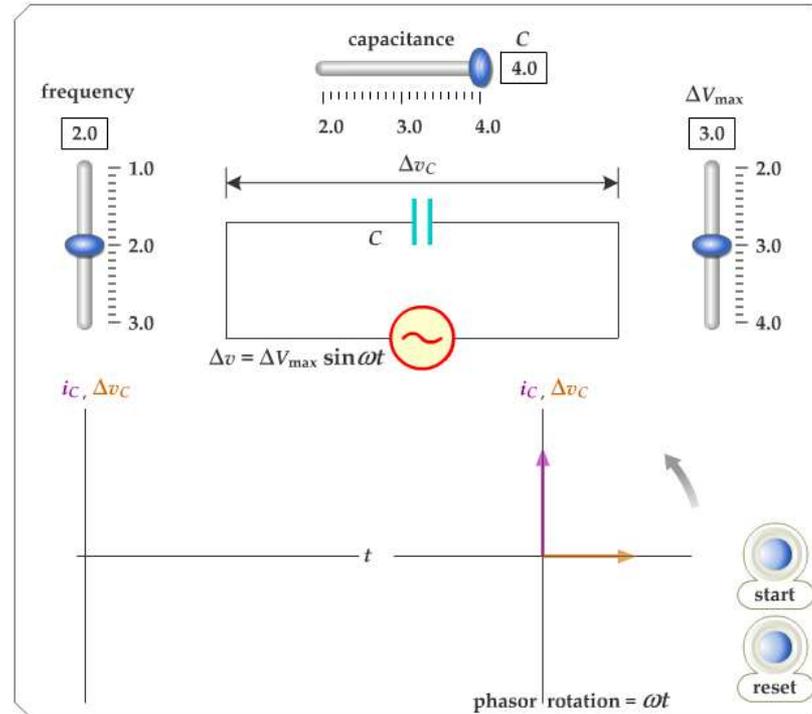


(b)



Active Figure 33.9

- Use the active figure to adjust capacitance, frequency and voltage
- Observe the effect on the graph and the phasor diagram



**PLAY
ACTIVE FIGURE**



Capacitive Reactance

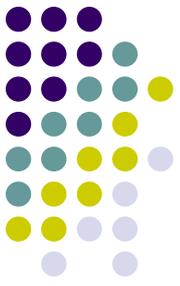
- The maximum current in the circuit occurs at $\cos \omega t = 1$ which gives

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{(1/\omega C)}$$

- The impeding effect of a capacitor on the current in an AC circuit is called the **capacitive reactance** and is given by

$$X_C \equiv \frac{1}{\omega C} \quad \text{which gives} \quad I_{\max} = \frac{\Delta V_{\max}}{X_C}$$

Voltage Across a Capacitor

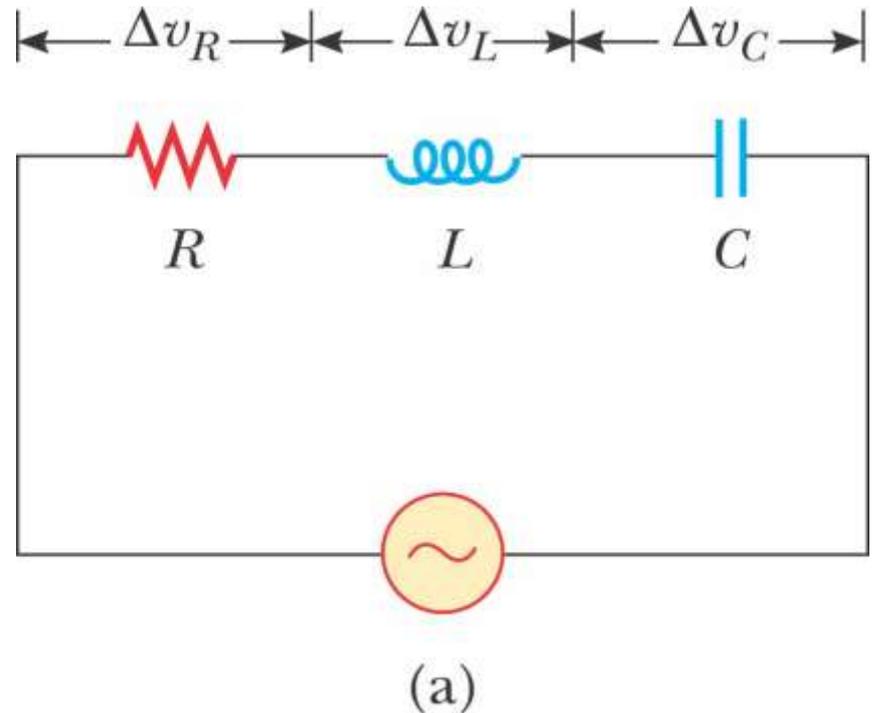


- The instantaneous voltage across the capacitor can be written as $\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t$
- As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases
- As the frequency approaches zero, X_C approaches infinity and the current approaches zero
 - This would act like a DC voltage and the capacitor would act as an open circuit

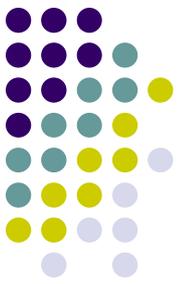


The *RLC* Series Circuit

- The resistor, inductor, and capacitor can be combined in a circuit
- The current and the voltage in the circuit vary sinusoidally with time



The *RLC* Series Circuit, cont.

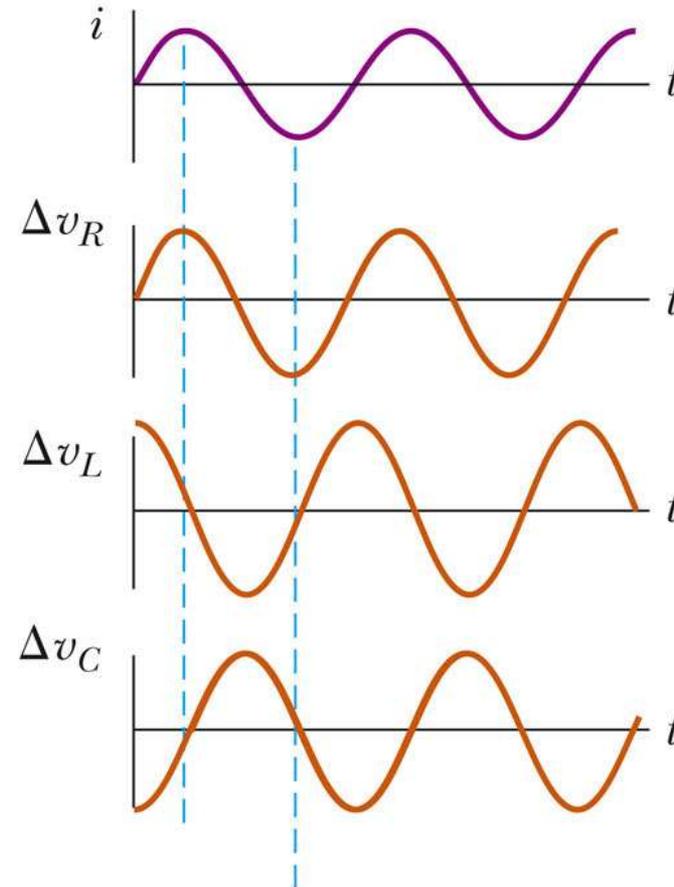


- The instantaneous voltage would be given by
$$\Delta v = \Delta V_{max} \sin \omega t$$
- The instantaneous current would be given by
$$i = I_{max} \sin (\omega t - \varphi)$$
 - φ is the **phase angle** between the current and the applied voltage
- Since the elements are in series, the current at all points in the circuit has the same amplitude and phase

i and v Phase Relationships – Graphical View



- The instantaneous voltage across the resistor is in phase with the current
- The instantaneous voltage across the inductor leads the current by 90°
- The instantaneous voltage across the capacitor lags the current by 90°



(b)

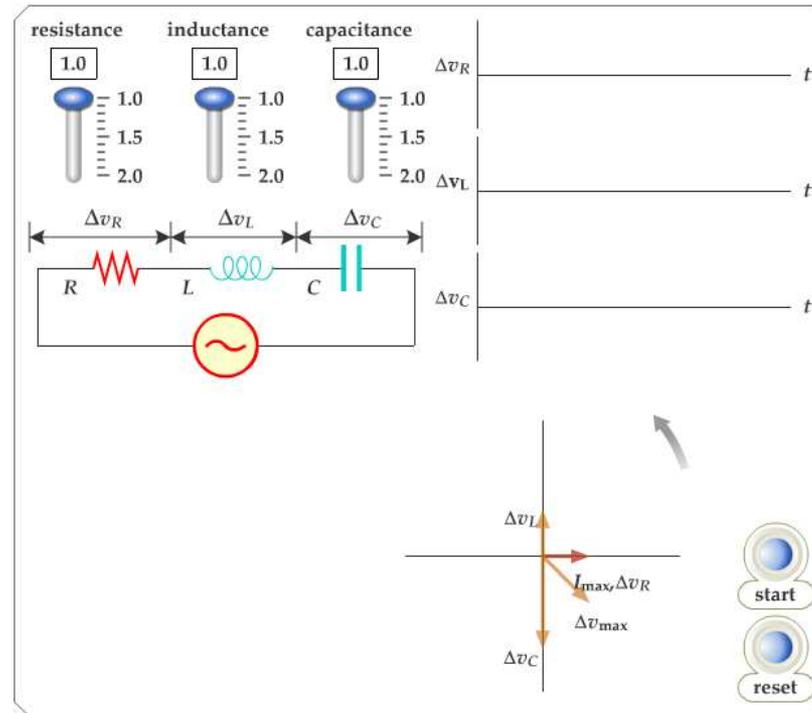
© Thomson Higher Education

PLAY
ACTIVE FIGURE



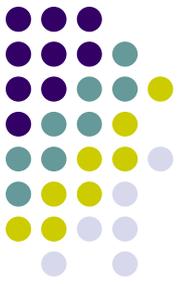
Active Figure 33.13

- Use the active figure to adjust the resistance, capacitance and inductance
- Observe the results on the graph and the phasor diagram



**PLAY
ACTIVE FIGURE**

i and *v* Phase Relationships – Equations



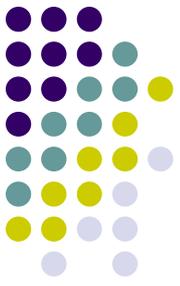
- The instantaneous voltage across each of the three circuit elements can be expressed as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{\max} X_L \sin \left(\omega t + \frac{\pi}{2} \right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{\max} X_C \sin \left(\omega t - \frac{\pi}{2} \right) = -\Delta V_C \cos \omega t$$

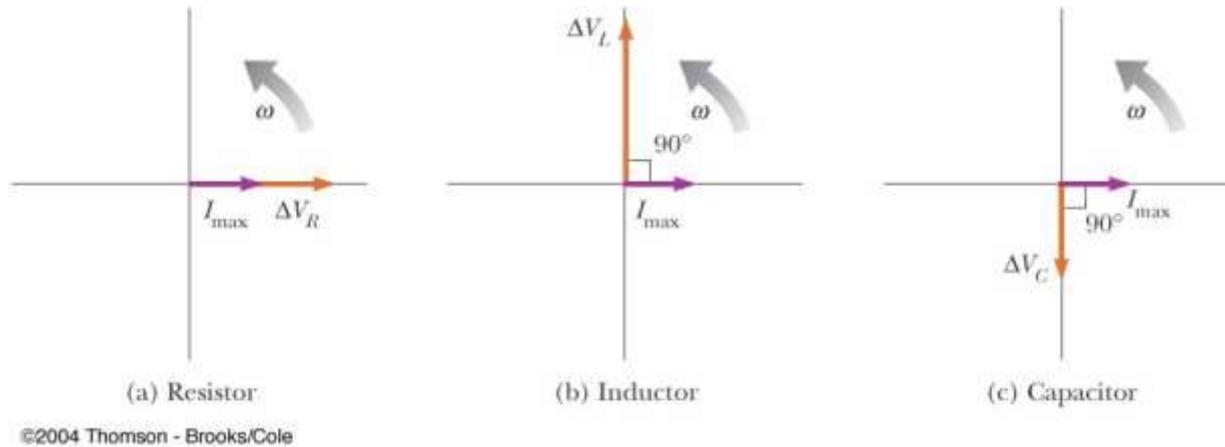
More About Voltage in RLC Circuits



- ΔV_R is the maximum voltage across the resistor and $\Delta V_R = I_{\max} R$
- ΔV_L is the maximum voltage across the inductor and $\Delta V_L = I_{\max} X_L$
- ΔV_C is the maximum voltage across the capacitor and $\Delta V_C = I_{\max} X_C$
- The sum of these voltages must equal the voltage from the AC source
- Because of the different phase relationships with the current, they cannot be added directly



Phasor Diagrams

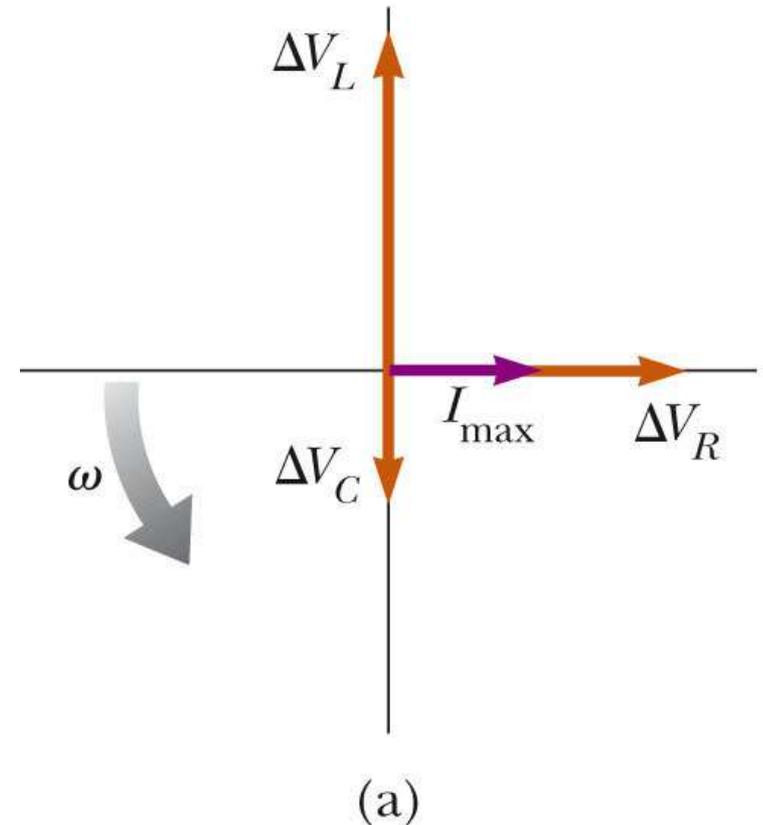


- To account for the different phases of the voltage drops, vector techniques are used
- Remember the phasors are rotating vectors
- The phasors for the individual elements are shown



Resulting Phasor Diagram

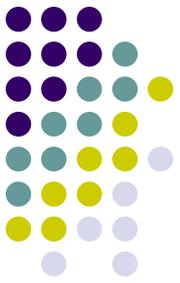
- The individual phasor diagrams can be combined
- Here a single phasor I_{\max} is used to represent the current in each element
 - In series, the current is the same in each element



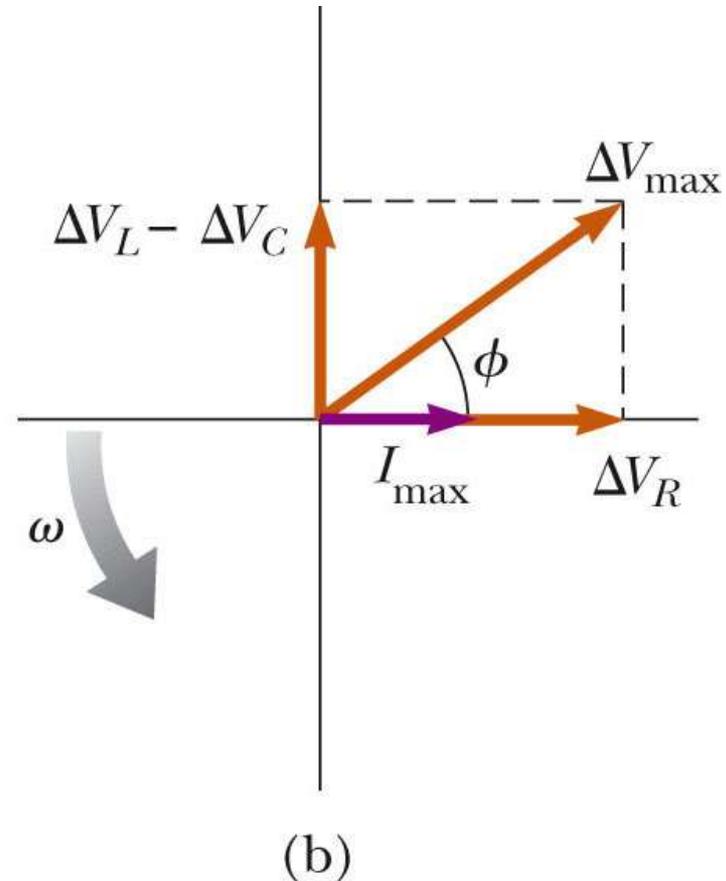
© Thomson Higher Education

PLAY
ACTIVE FIGURE

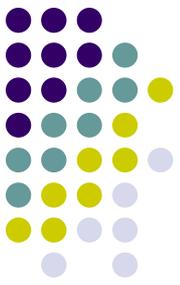
Vector Addition of the Phasor Diagram



- Vector addition is used to combine the voltage phasors
- ΔV_L and ΔV_C are in opposite directions, so they can be combined
- Their resultant is perpendicular to ΔV_R



PLAY
ACTIVE FIGURE



Total Voltage in RLC Circuits

- From the vector diagram, ΔV_{\max} can be calculated

$$\begin{aligned}\Delta V_{\max} &= \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} \\ &= \sqrt{(I_{\max} R)^2 + (I_{\max} X_L - I_{\max} X_C)^2} \\ \Delta V_{\max} &= I_{\max} \sqrt{R^2 + (X_L - X_C)^2}\end{aligned}$$



Impedance

- The current in an RLC circuit is

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{\max}}{Z}$$

- Z is called the impedance of the circuit and it plays the role of resistance in the circuit, where

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

- Impedance has units of ohms



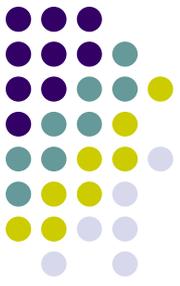
Phase Angle

- The right triangle in the phasor diagram can be used to find the phase angle, φ

$$\varphi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

- The phase angle can be positive or negative and determines the nature of the circuit

Determining the Nature of the Circuit



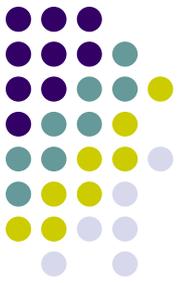
- If ϕ is positive
 - $X_L > X_C$ (which occurs at high frequencies)
 - The current lags the applied voltage
 - The circuit is *more inductive than capacitive*
- If ϕ is negative
 - $X_L < X_C$ (which occurs at low frequencies)
 - The current leads the applied voltage
 - The circuit is *more capacitive than inductive*
- If ϕ is zero
 - $X_L = X_C$
 - The circuit is *purely resistive*



Power in an AC Circuit

- The average power delivered by the AC source is converted to internal energy in the resistor
 - $\mathcal{P}_{av} = \frac{1}{2} I_{max} \Delta V_{max} \cos \phi = I_{rms} \Delta V_{rms} \cos \phi$
 - $\cos \phi$ is called the power factor of the circuit
- We can also find the average power in terms of R
 - $\mathcal{P}_{av} = I_{rms}^2 R$

Power in an AC Circuit, cont.



- The average power delivered by the source is converted to internal energy in the resistor
- No power losses are associated with pure capacitors and pure inductors in an AC circuit
 - In a capacitor, during one-half of a cycle, energy is stored and during the other half the energy is returned to the circuit and no power losses occur in the capacitor
 - In an inductor, the source does work against the back emf of the inductor and energy is stored in the inductor, but when the current begins to decrease in the circuit, the energy is returned to the circuit



Power and Phase

- The power delivered by an AC circuit depends on the phase
- Some applications include using capacitors to shift the phase to heavy motors or other inductive loads so that excessively high voltages are not needed



Resonance in an AC Circuit

- *Resonance* occurs at the frequency ω_o where the current has its maximum value
 - To achieve maximum current, the impedance must have a minimum value
 - This occurs when $X_L = X_C$
 - Solving for the frequency gives

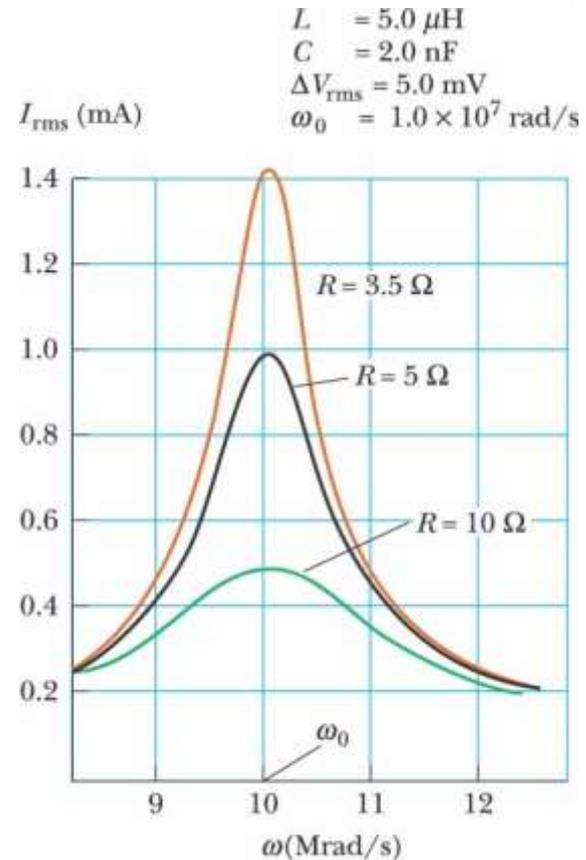
$$\omega_o = \frac{1}{\sqrt{LC}}$$

- The resonance frequency also corresponds to the natural frequency of oscillation of an LC circuit



Resonance, cont.

- Resonance occurs at the same frequency regardless of the value of R
- As R decreases, the curve becomes narrower and taller
- Theoretically, if $R = 0$ the current would be infinite at resonance
 - Real circuits always have some resistance



(a)

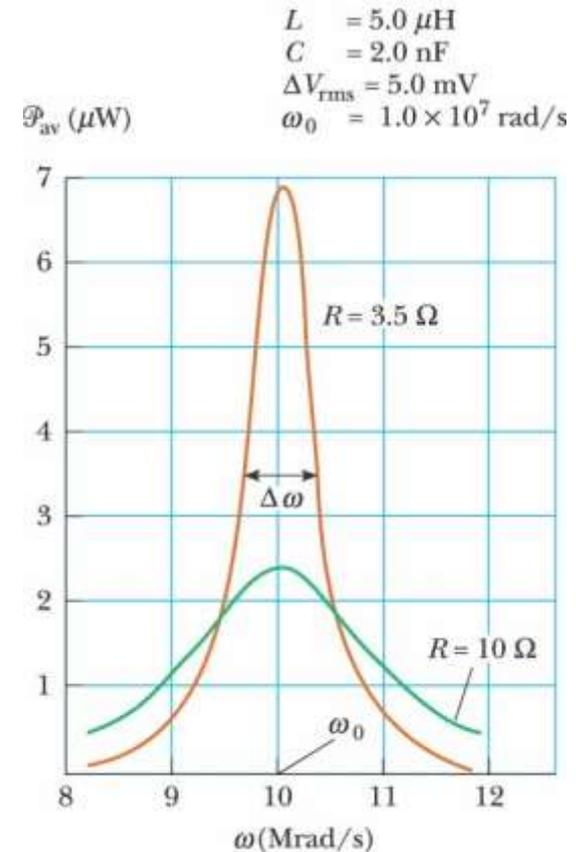
Power as a Function of Frequency



- Power can be expressed as a function of frequency in an RLC circuit

$$\mathcal{P}_{av} = \frac{(\Delta V_{rms})^2 R \omega^2}{R^2 \omega^2 + L^2 (\omega^2 - \omega_0^2)^2}$$

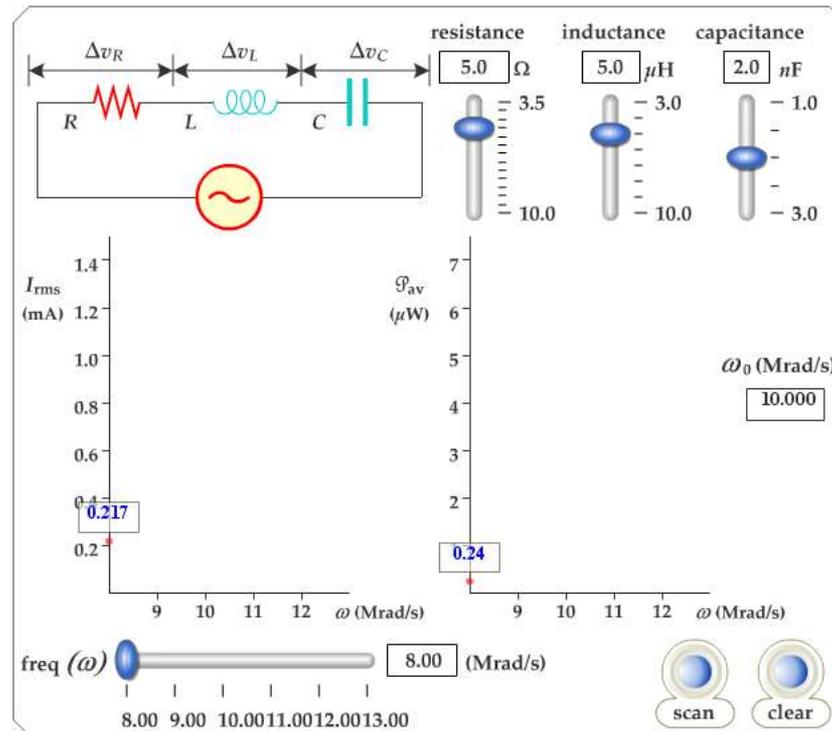
- This shows that **at resonance, the average power is a maximum**





Active Figure 33.17

- Use the active figure to adjust R, L and C in the circuit
- Determine the current and the power

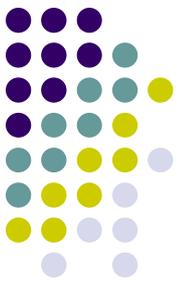


**PLAY
ACTIVE FIGURE**



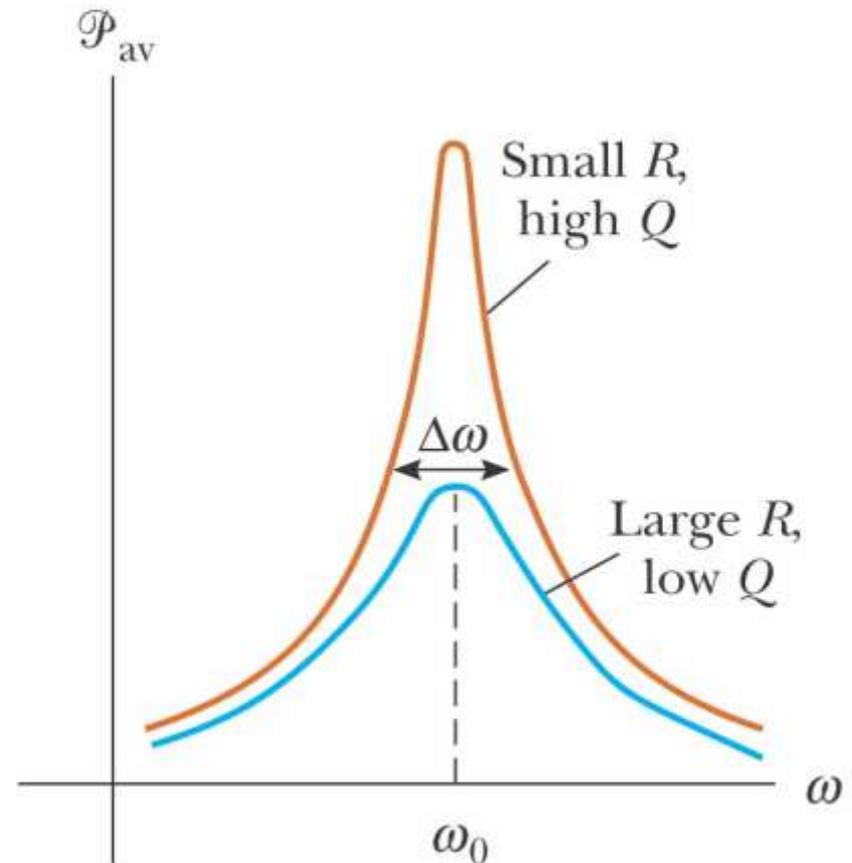
Quality Factor

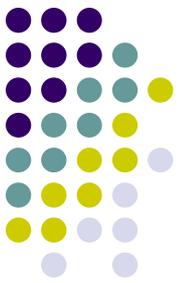
- The sharpness of the resonance curve is usually described by a dimensionless parameter known as the quality factor, Q
- $Q = \omega_0 / \Delta\omega = (\omega_0 L) / R$
 - $\Delta\omega$ is the width of the curve, measured between the two values of ω for which ϕ_{avg} has half its maximum value
 - These points are called the *half-power points*



Quality Factor, cont.

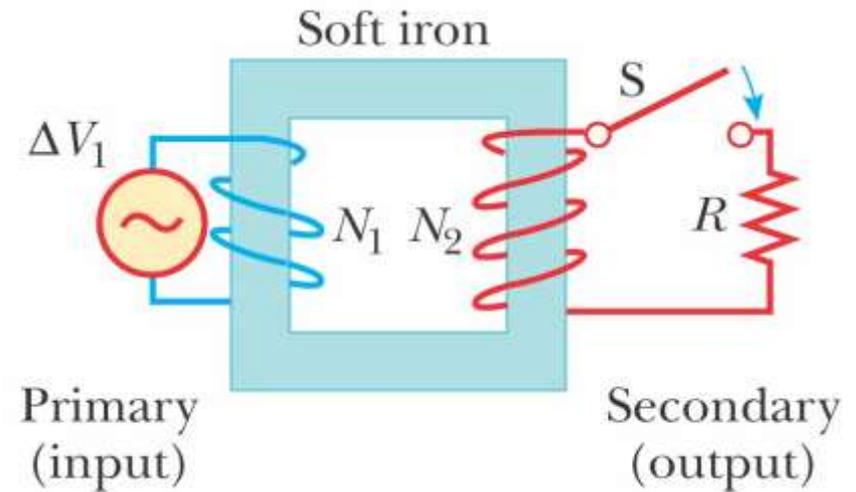
- A high- Q circuit responds only to a narrow range of frequencies
 - Narrow peak
- A low- Q circuit can detect a much broader range of frequencies





Transformers

- An **AC transformer** consists of two coils of wire wound around a core of iron
- The side connected to the input AC voltage source is called the *primary* and has N_1 turns





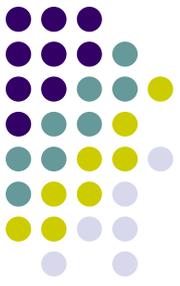
Transformers, 2

- The other side, called the secondary, is connected to a resistor and has N_2 turns
- The core is used to increase the magnetic flux and to provide a medium for the flux to pass from one coil to the other
 - Eddy-current losses are minimized by using a laminated core



Transformers, 3

- Assume an ideal transformer
 - One in which the energy losses in the windings and the core are zero
 - Typical transformers have power efficiencies of 90% to 99%
- In the primary, $\Delta V_1 = -N_1 \frac{d\Phi_B}{dt}$
- The rate of change of the flux is the same for both coils



Transformers, 4

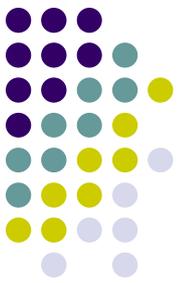
- The voltage across the secondary is

$$\Delta V_2 = -N_2 \frac{d\Phi_B}{dt}$$

- The voltages are related by

$$\Delta V_2 = \frac{N_2}{N_1} \Delta V_1$$

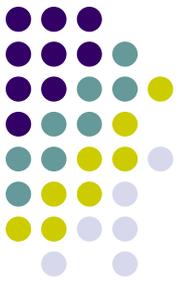
- When $N_2 > N_1$, the transformer is referred to as a step-up transformer
- When $N_2 < N_1$, the transformer is referred to as a step-down transformer



Transformers, 5

- The power input into the primary equals the power output at the secondary
 - $I_1 \Delta V_1 = I_2 \Delta V_2$
- The equivalent resistance of the load resistance when viewed from the primary is

$$R_{\text{eq}} = \left(\frac{N_1}{N_2} \right)^2 R_L$$



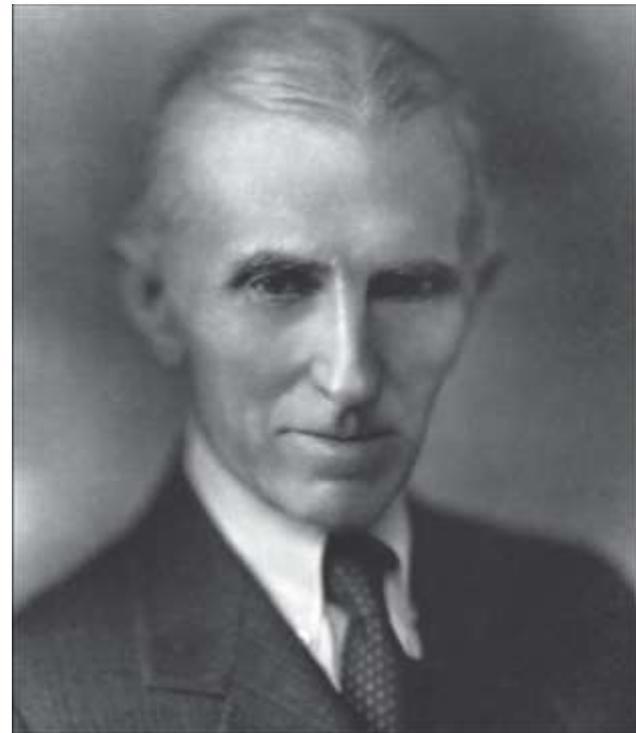
Transformers, final

- A transformer may be used to match resistances between the primary circuit and the load
- This way, maximum power transfer can be achieved between a given power source and the load resistance
 - In stereo terminology, this technique is called *impedance matching*



Nikola Tesla

- 1856 – 1943
- American physicist
- Key figure in development of
 - Alternating-current electricity
 - High-voltage transformers
 - Transport of electric power using AC transmission lines



© Thomson Higher Education

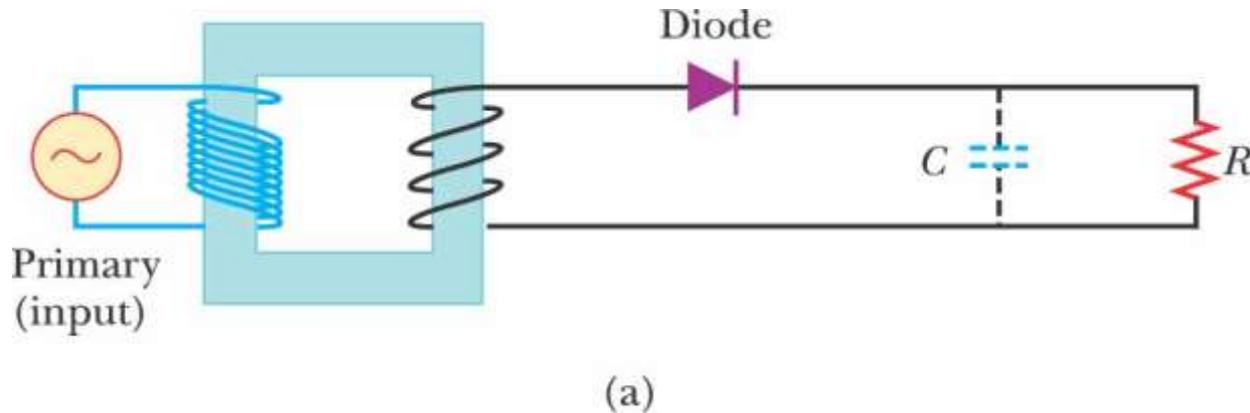


Rectifier

- The process of converting alternating current to direct current is called **rectification**
- A **rectifier** is the converting device
- The most important element in a rectifier circuit is the diode
 - A diode is a circuit element that conducts current in one direction but not the other

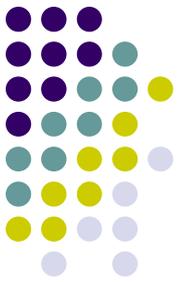


Rectifier Circuit



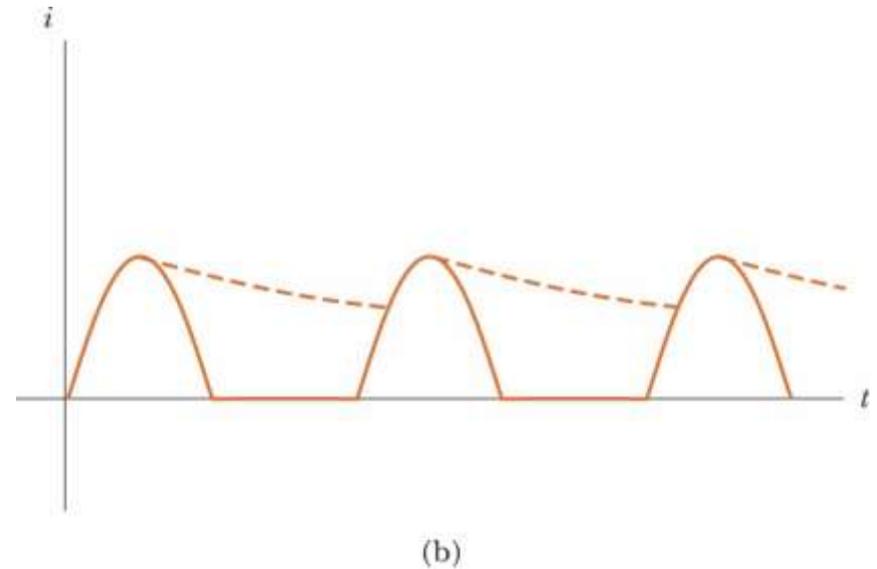
©2004 Thomson - Brooks/Cole

- The arrow on the diode (\rightarrow) indicates the direction of the current in the diode
 - The diode has low resistance to current flow in this direction
- Because of the diode, the alternating current in the load resistor is reduced to the positive portion of the cycle



Half-Wave Rectifier

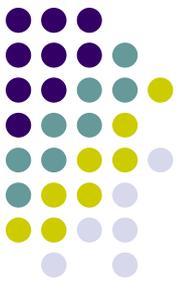
- The solid line in the graph is the result through the resistor
- It is called a *half-wave rectifier* because current is present in the circuit during only half of each cycle



Half-Wave Rectifier, Modification

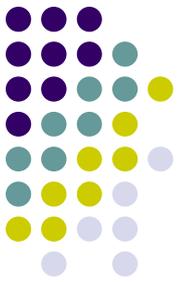


- A capacitor can be added to the circuit
- The circuit is now a simple DC power supply
- The time variation in the circuit is close to zero
 - It is determined by the RC time constant of the circuit
 - This is represented by the dotted lines in the previous graph



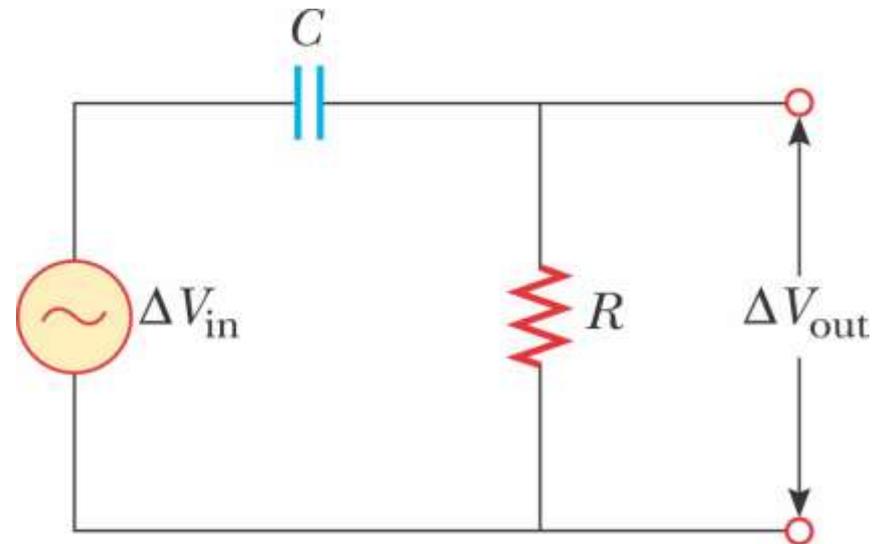
Filter Circuit, Example

- A **filter circuit** is one used to smooth out or eliminate a time-varying signal
- After rectification, a signal may still contain a small AC component
 - This component is often called a ripple
- By filtering, the ripple can be reduced
- Filters can also be built to respond differently to different frequencies

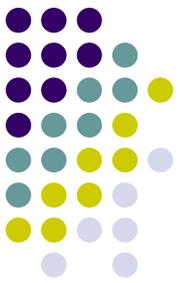


High-Pass Filter

- The circuit shown is one example of a **high-pass filter**
- A high-pass filter is designed to preferentially pass signals of higher frequency and block lower frequency signals

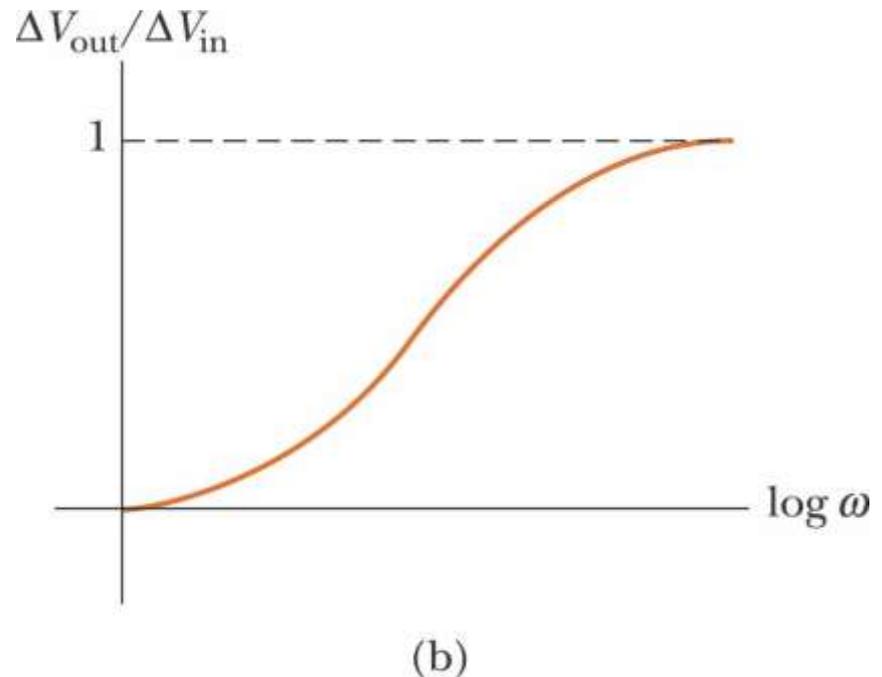


(a)



High-Pass Filter, cont

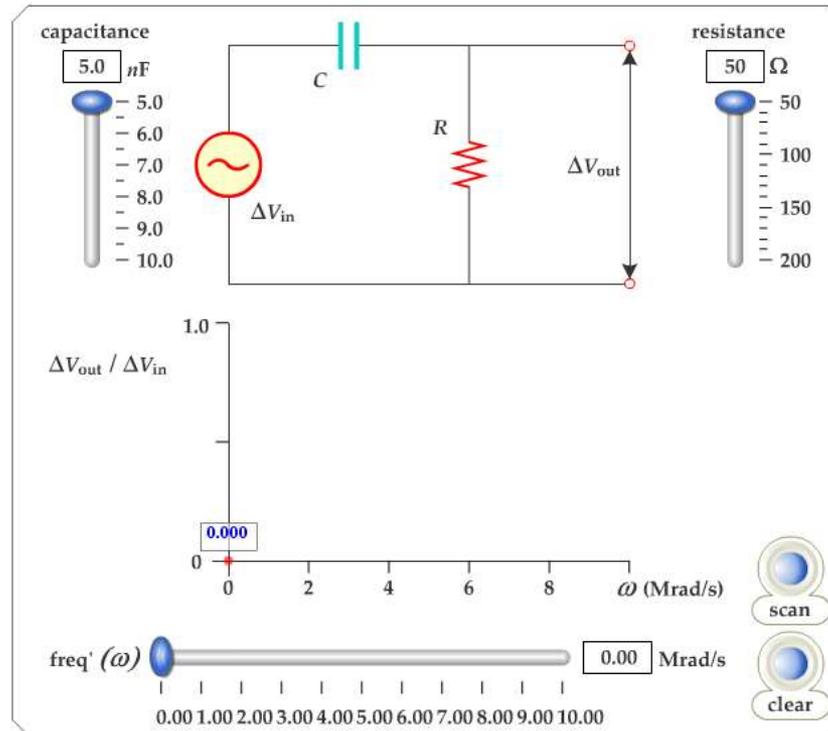
- At low frequencies, ΔV_{out} is much smaller than ΔV_{in}
 - At low frequencies, the capacitor has high reactance and much of the applied voltage appears across the capacitor
- At high frequencies, the two voltages are equal
 - At high frequencies, the capacitive reactance is small and the voltage appears across the resistor



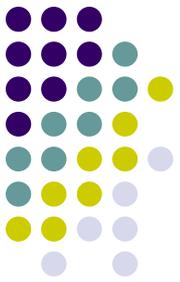


Active Figure 33.25

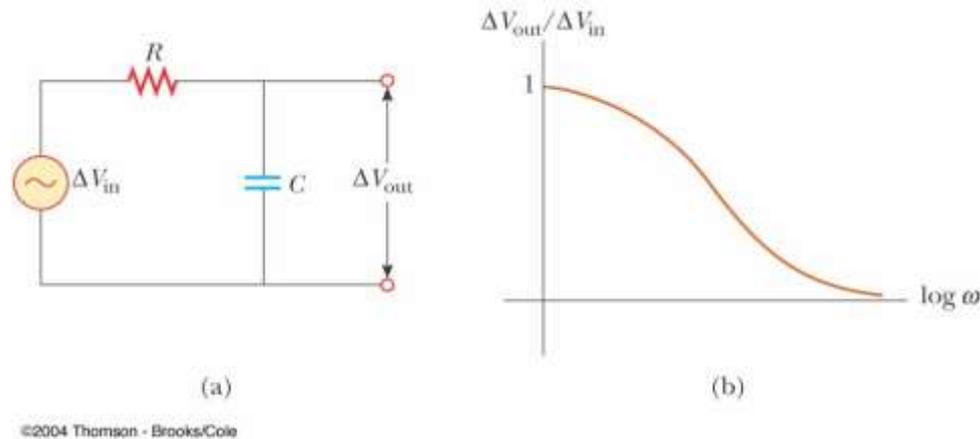
- Use the active figure to adjust C and R
- Determine the output voltage for a given frequency
- Sweep through the frequencies and observe the results



**PLAY
ACTIVE FIGURE**



Low-Pass Filter

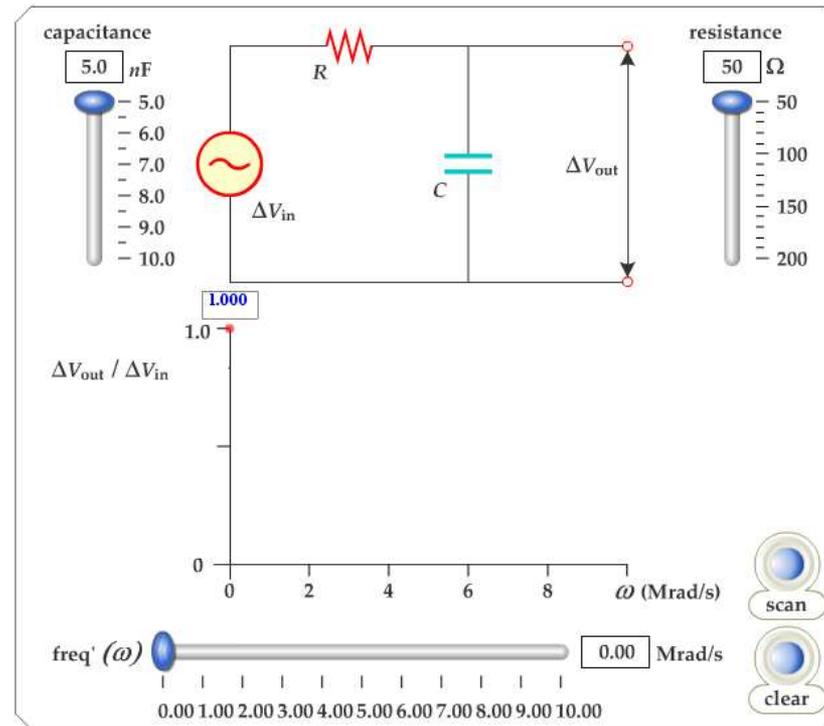


- At low frequencies, the reactance and voltage across the capacitor are high
- As the frequency increases, the reactance and voltage decrease
- This is an example of a low-pass filter



Active Figure 33.23

- Use the active figure to adjust R and C
- Determine the output voltage for a given frequency
- Sweep through the frequencies and observe the results



**PLAY
ACTIVE FIGURE**