Arab Academy for Science and Technology & Maritime Transport
College of Engineering and Technology
Department of Basic and Applied Sciences

MATH 1

- *Rules of differentiation*
- *Trigonometric functions and their derivatives*
- *Inverse trigonometric functions and their derivatives*
- *Logarithmic function and its derivative*
- *Exponential function and its derivative*
- *Derivatives of Hyperbolic and inverse Hyperbolic functions*
- *Parametric function and its derivative*
- *Implicit function and its derivative*
- *L'Hôpital's rule (The limit of a function)*
- *Maclaurin's expansions*
- *Partial differentiation*
- *Curve sketching*
- *Physical application (velocity and acceleration)*
- *Conic sections (Parabola – Ellipse – Hyperbola)*
- *Software application*

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Syllabus for Mathematics 1 (BA123)
Text Book: Calculus,
Sherman K. Stein & Anthony Barcellos
Program title: All Programs
Coordinator: Prof. Nasser El-Maghraby (R. 124)

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13	Conic section : Parabola Equation, Vertex, Focus, Directrix, Eccentricity, Graph	Sheet 12	
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Sheet 1 : Basic Differentiation Rules

	y = y(x)	y'=dy/dx=y'(x)
1.	$y = k$,k: constant	$y' = 0$
2.	$y = x^k$,k is constant	$y' = kx^{k-1}$
3.	$y = f(x) \pm g(x)$	$y' = f'(x) \pm g'(x)$
4.	$y = kf(x)$,k is constant	$y' = kf'(x)$
5.	$y = f(x)\, g(x)$	$y' = f(x)g'(x) + f'(x)g(x)$
6.	$y = [f(x)]^k$,k is constant	$y' = k[f(x)]^{k-1} f'(x)$
7.	$y = f(x)/g(x)$	$y' = [f'(x)g(x) - f(x)g'(x)]/g^2(x)$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = x^4 - 3x^{-2} + 15x + 10$

2) $y = (\sqrt{x} - 1)^7$

3) $y = \dfrac{1}{(x^6 - 2)^5}$

4) $y = (x^3 - 1)^5 (2 + 3x^{-4})^7$

5) $y = \dfrac{x^3 - 1}{x^3 + 1}$

6) $y = \left(\dfrac{x^2 - 3}{x^{-4} + 2}\right)^{4/3}$

b. Find d^2y/dx^2 for each of the following

1) $y = x^7 - \dfrac{2}{x^3} + x^{-5} + 16x + 5$

2) $y = (2 - x^3)^8$

c. If $y = (x + \sqrt{x^2 - 1})^4$**, Show that** $y' = \dfrac{4y}{\sqrt{x^2 - 1}}$

Classroom Exercises

d. Find dy/dx for each of the following

1) $y=\frac{3}{x}-\frac{4}{x^2}+6x^5+7$

2) $y=(x^4-1)^6$

3) $y=\left(\frac{1-x^4}{1+x^4}\right)^{3/2}$

4) $y=\sqrt{x^3-1}\,(1-3x)^5$

5) $y=\sqrt{\frac{x-1}{x+1}}$

6) $y=\left(1-\frac{1}{\sqrt{x}}\right)^{-4/3}$

e. Find d^2y/dx^2 for each of the following

1) $y=(x^3-1)^6$

2) $y=\frac{x^2-1}{x^2+1}$

Homework

f. Find dy/dx for each of the following

1) $y=\sqrt{x^5-4}$

2) $y=x^{-3}(1+x^4)^5$

3) $y=(x^2+4)^6(1-2x)^7$

4) $y=\sqrt{x}\,(1-\sqrt{x})^6$

5) $y=(\sqrt{1+x^2})^5\sqrt[3]{x^4-1}$

6) $y=\left(\frac{x^2-4}{x^2+2}\right)^{7/2}$

g. Find d^2y/dx^2 for each of the following

1) $y=(x^{3/2}-1)^4$

2) $y=\frac{x^2-1}{\sqrt{x+1}}$

Sheet 2 : Trigonometric Functions and their Derivatives

$\cos^2 u + \sin^2 u = 1$

$1 + \tan^2 u = \sec^2 u$ $\qquad$ $y = \cos u \longrightarrow y' = -u' \sin u$

$\cot^2 u + 1 = \text{cosec}^2 u$ $\qquad$ $y = \sin u \longrightarrow y' = u' \cos u$

$\sin 2u = 2 \sin u \cos u$ $\qquad$ $y = \tan u \longrightarrow y' = u' \sec^2 u$

$\cos 2u = \cos^2 u - \sin^2 u$ $\qquad$ $y = \cot u \longrightarrow y' = -u' \text{cosec}^2 u$

$= 2\cos^2 u - 1$ $\qquad$ $y = \sec u \longrightarrow y' = u' \sec u \tan u$

$= 1 - 2\sin^2 u$ $\qquad$ $y = \text{cosec}\, u \longrightarrow y' = -u' \text{cosec}\, u \cot u$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = \sin x^3$

2) $y = (1 + \cos^3 x)\cot^2 2x$

3) $y = x^3 \cos x^2 - 2\cot x^{-3}$

4) $y = \dfrac{x \sin 2x}{1 - \cos^2 3x}$

5) $y = \sec^3 \sqrt{4x^2 + 1}$

6) $y = \dfrac{\sin(x-1)}{x-1}$

b. Find d^2y/dx^2 for each of the following

1) $y = (1 - \cos^2 x)^{-3/2}$

2) $y = x \sec x$

c. If y = a sin ct + b cos ct , where a,b and c are constants

prove that $y'' = -c^2 y$

Classroom Exercises

d. Find dy/dx for each of the following

1) $y = \tan^2(\cos^3 x^2)$

2) $y = \sqrt{x^2 + 1}\cos^3\sqrt{x^2 - 1}$

3) $y = x^2 \sec^3 x - 4\cot^2 x^3$

4) $y = \dfrac{1 - \sin 2x}{1 + \cos 2x}$

5) $y = \sqrt{\tan^2 x + x \cos^3 x}$

6) $y = \sqrt{x - 1}\sin\sqrt{x - 1}$

e. **Find d^2y/dx^2 for each of the following**

1) $y = x^4(\cos 2x)$ 2) $y = \dfrac{\sin x}{x}$

Homework

f. Find dy/dx for each of the following

1) $y = \sec^3\sqrt{\cos x}$ 2) $y = \cot(\sqrt{x}\tan\sqrt{x})$

3) $y = x^{3/2}\cot x^3$ 4) $y = \operatorname{cosec}^4\sqrt{x^2 - 1}$

5) $y = (1 - \sin\sqrt{x})^3\cos\sqrt{x}$ 6) $y = \sqrt{x}\operatorname{cosec}\sqrt{x}$

g. Find d^2y/dx^2 for each of the following

1) $y = x\tan x^3$ 2) $y = \sin^3 x$

Sheet 3: Inverse Trigonometric Functions and their Derivatives

$$y = \sin^{-1}u \rightarrow y' = \frac{u'}{\sqrt{1-u^2}} \qquad y = \cos^{-1}u \rightarrow y' = \frac{-u'}{\sqrt{1-u^2}}$$

$$y = \tan^{-1}u \rightarrow y' = \frac{u'}{1+u^2} \qquad y = \cot^{-1}u \rightarrow y' = \frac{-u'}{1+u^2}$$

$$y = \sec^{-1}u \rightarrow y' = \frac{u'}{|u|\sqrt{u^2-1}} \qquad y = \operatorname{cosec}^{-1}u \rightarrow y' = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = \cos^{-1}\sqrt{x}$ **2)** $y = (x^2 + 4)\operatorname{cosec}^{-1}2x$

3) $y = x^3(1 - \sec^{-1}x)$ **4)** $y = x^3\sin^{-1}\sqrt{x} - 2\cot^{-1}x^2$

5) $y = \tan^{-1}\left(\dfrac{x-1}{x+1}\right)$ **6)** $y = \dfrac{1-\sin^{-1}x}{\cos^{-1}x}$

b. If y = tan(cos $^{-1}$x), Prove that $y' = \dfrac{-(y^2+1)}{\sqrt{1-x^2}}$

Classroom Exercises

c. Find dy/dx for each of the following

1) $y = \sqrt{x}\tan^{-1}\sqrt{x}$

2) $y = \cot^{-1}\left(\dfrac{\cos 3x}{1+\sin 3x}\right)$

3) $y = x^3 \sec^{-1} x^2$

4) $y = \dfrac{\cos^{-1} x}{1-\sin^{-1} x}$

5) $y = \sqrt{x^2-1}\sin^{-1} x - x\cos^{-1} x$

6) $y = \tan^{-1}(\cos x) + \cot^{-1}(\sin x)$

d. If y = cos(2sin^{-1}x), prove that $(1-x^2)(y')^2 = 4(1-y^2)$

Homework

e. Find dy/dx for each of the following

1) $y = \sin^{-1} x^3$

2) $y = \cot^{-1}(cos\, 2x)$

3) $y = \cot^{-1}\left(\dfrac{x-4}{x+4}\right)$

4) $y = \dfrac{\tan^{-1} x}{1-\cos^{-1}\sqrt{x}}$

5) $y = x^2 \cos ec^{-1}\sqrt{x} - 3x\sin^{-1} x$

6) $y = \sqrt[3]{x}\sec^{-1}\left(\dfrac{x}{4}\right)$

f. Prove that $\dfrac{d}{dx}\left(\tan^{-1}\left(\dfrac{x-1}{x+1}\right)\right) = \dfrac{d}{dx}\left(\tan^{-1}(x)\right)$

Sheet 4 : Logarithmic Function and Its Derivatives

$\ln b = a \Leftrightarrow b = e^a$, e = 2.71828 .....,b ≥ 0

1. **ln(1) = 0, ln(0) = - ∞, ln(*e*) = 1**
2. **ln(*ab*) = ln a + ln b**
3. **ln(a^n) = n ln a**
4. **ln (e^n) = n**
5. **ln *(a/b)* = ln *a* – ln *b***

$$y = \ln u \rightarrow y' = \frac{u'}{u}$$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = x^3 \ln x$

2) $y = \ln(x^{-4}(x^5 - 2)^6)$

3) $y^x = x^y$

4) $y = \ln\left[\dfrac{x^3(1-x^2)^4}{\sin x\,(x-1)^5}\right]^{7/2}$

5) $y = \sin x^2 - 3x \cos x - x^x$

6) $(\sin x)^{\cos y} = (\sin y)^{\cos x}$

b. If $y = \ln(\sec x + \tan x)$**, Prove that** $y'' = \sec x \tan x$ **.**

Classroom Exercises

c. Find dy/dx for each of the following

1) $y = (\ln x)^3$

2) $y = \ln\left(\dfrac{x^3 - 1}{x^2 + 1}\right)$

3) $y = \sqrt[4]{\dfrac{(1-x)^3 \tan^{-1} x}{x^x \sec x^3}}$

4) $y^{5/2} = x^{\ln x}$

5) $y = \dfrac{x^x (2 - \sin x)^{x^2}}{x^{\cos x}(1 - 2\ln x)^5}$

6) $y = x \cos\sqrt{x} - x^{\sec x}$

d. If $y = \cos(\ln x) + \sin(\ln x)$ **prove that** $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

Homework

e. Find dy/dx for each of the following

1) $y = \ln(1 - \ln x)$

2) $y = \ln\left[\dfrac{(1-x^2)^5(2 - \sin^{-1} x)}{(1 - \ln x)^2(3 - \cos x)}\right]$

3) $y = \dfrac{(x-1)^3(1 - \sin x)^4}{x^x(2 - \cos x)^2}$

4) $\sqrt{y} = \dfrac{x^5 \tan^{-1} x}{(1+x)\sqrt[3]{x}}$

5) $y = x^{\sin x}$

6) $y = (\ln(\sin x))^{\cos x}$

Sheet 5 : Exponential Function and Its Derivative

1. $e^a e^b = e^{a+b}$
2. $e^a / e^b = e^{a-b}$
3. $(e^a)^b = e^{ab}$
4. $e^{\ln a} = a$

$$y = e^u \rightarrow y' = u' e^u$$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = e^{\sin^{-1} x}$

2) $y = e^{\tan^{-1} \sin x}$

3) $y = \cos^3 e^{x^2}$

4) $y = \cos^{-1}(1 - e^{-x})$

5. $y = \operatorname{cosec}^{-1} e^x - x^4 e^{\cot x}$

6. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

b. If $y = \dfrac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$, show that $y' = \dfrac{8}{\left(e^{2x} + e^{-2x}\right)^2}$.

c. If $y = \tan^{-1} \ln e^{\tan \sqrt{x}}$, show that yy’ = 1/ 2

Classroom Exercises

d. Find dy/dx for each of the following

1) $y = x^3 e^{x^5 - 3}$

2) $y = e^{\ln \sin^{-1}(\sin x3)}$

3) $y = \sqrt{e^{\cos^{-1} x}}$

4) $y = \ln\left[\dfrac{e^{x^2} \sin x^3}{(1 - e^x)(2 - x)}\right]^6$

e. If $y = \ln(\cos x)$**, show that** $y'' + e^{-2y} = 0$

f. Find d^2y/dx^2 **for each of the following**

1) $y = e^{\sin x}$

2) $y = \cos e^{3x}$

Homework

g. Find dy/dx for each of the following

1) $y = e^{x^3}$

2) $y = e^{\cos^{-1}(\sin x)}$

3) $y = \ln\left[\dfrac{1+e^{2x}}{(1-e^{-2x})^3}\right]$

4) $y = x^5 \sec e^{-x}$

h. If $y = ae^{-2x} + be^{3x}$ **where a and b are constant**

Show that

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$$

i. Find d^2y/dx^2 for each of the following

1) $y = e^{-4x}$

2) $y = \ln(e^{2x} - 4)$

Sheet 6 : Derivatives of Hyperbolic Functions and Their Inverse

$\sinh x = \dfrac{e^x - e^{-x}}{2}$ $\qquad \cosh x = \dfrac{e^x + e^{-x}}{2}$ $\qquad \tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$\operatorname{cosech} x = \dfrac{2}{e^x - e^{-x}}$ $\qquad \operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$ $\qquad \coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

$\cosh^2 u - \sinh^2 u = 1$

$1 - \tanh^2 u = \sec h^2 u$

$\coth^2 u - 1 = \operatorname{cosech}^2 u$

$\sinh 2u = 2\sinh u \cosh u$

$\cosh 2u = \cosh^2 u + \sinh^2 u$

$\tan 2u = 2\tanh u / (1 + \tanh^2 u)$

$y = \sinh u \rightarrow y' = u' \cosh u$

$y = \cosh u \mapsto \qquad ' \sinh u$

$y = \tanh u \rightarrow y' = u' \operatorname{sech}^2 u$

$y = \coth u \rightarrow y' = -u' \operatorname{cosech}^2 u$

$y = \operatorname{sech} u \mapsto \qquad u' \operatorname{sech} u \tanh u$

$y = \operatorname{cosech} u \rightarrow y' = -u' \operatorname{cosech} u \coth u$

$y = \sinh^{-1} u \rightarrow y' = \dfrac{u'}{\sqrt{u^2+1}}$

$y = \cosh^{-1} u \rightarrow y' = \dfrac{u'}{\sqrt{u^2-1}}$

$y = \tanh^{-1} u \rightarrow y' = \dfrac{u'}{1-u^2}$, $|u| \prec$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = x^4 \cosh^2 x^3$

2) $y = \tanh(x \ln x)$

3) $y = e^{\cosh^{-1} x^2}$

4) $y = (\sin^{-1} \sqrt{x})(1 - \cosh^{-1} x^2)$

5) $y = \sqrt[5]{x^3} \tanh^{-1} x^2$

6) $y = \ln\left[\dfrac{(x+1)^2 e^{\operatorname{cosech} x}}{\sqrt{x^3 - 1}}\right]$

b. **Show that** $\cosh^{-1} x = \ln(x \pm \sqrt{x^2 - 1})$

Classroom Exercises

c. Find dy/dx for each of the following

1) $y = \sinh x^3$

2) $y = \tanh^{-1}(\operatorname{sec} h 2x)$

3) $y = \sin\left(\cosh^{-1} \sqrt{x^2 + 1}\right)$

4) $y = \sqrt{\cosh^{-1}(e^{-x/2})}$

d. Show that

$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$

e. Solve the following equations

1. $e^{\cosh^{-1} x} = 2$

2. $\ln\left(\dfrac{1 + \tanh x}{1 - \tanh x}\right) = 5$

Homework

f. Find dy/dx for each of the following

1. $y = x^2 \coth^3 \sqrt{x}$

2. $y = x e^{\sinh^{-1} x}$

3. $y = (1 - \ln \sec x) \cosh^{-1} \sqrt{x}$

4. $y = \ln \sqrt{\tanh 3x}$

5. $y = \sinh^{-1}(\sin 2x)$

6. $y = \tanh^{-1} \sqrt{\sec x}$

f. Show that

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$$

Sheet 7 : Parametric and Implicit Differentiation

Lecture Examples

a. Find dy/dx for each of the following

1) $y = t \ln t \quad , \quad y = \ln t / t$ **2)** $x = e^t \cosh t \quad , \quad y = e^t \sinh t$

b. Find d^2y/dx^2 for each of the following

1) $x = \operatorname{cosec} t \quad , \quad y = \cos 2t$ **2)** $x = \sqrt{1-t^2} \quad , \quad y = \sin^{-1} t$

c. If $x = \cos \frac{t}{1+t} \quad , \quad y = \sin \frac{t}{1+t}$, **show that** $y^3 y'' + 1 = 0$

d. If $x = \frac{t+1}{t-1} \quad , \quad y = \left(\frac{t-1}{t+1} \right)^5$, **show that** $y'' = 30x^{-7}$

e. Find dy/dx for each of the following

1) $x^3 - 3x^2y^4 + 7y^2 = 10$ **2)** $x + \cos^{-1} y = xy$

3) $\sin^{-1} x + \tan(xy) = 5$ **4)** $y = e^{-x} + e^y$

Classroom Exercises

f. Find dy/dx for each of the following

1) $x = \frac{3t}{1+t^3} \quad , \quad y = \frac{3t}{1+t^3}$ 2) $x = \sqrt{1 - \sin\theta} \quad , \quad y = \sqrt{1 + \cos\theta}$

g. Find d^2y/dx^2 for each of the following

1) $x = \cos\theta + \theta \sin\theta \quad , \quad y = \sin\theta - \theta\cos\theta$ 2) $x = \sqrt{t^4 - 1} \quad , \quad y = \sec^{-1} t^2$

h. If $x = \tan\dfrac{t-1}{1+t}$, $y = \sec\dfrac{t-1}{1+t}$, **show that** $y'' = y^{-3}$ **.**

i. Find dy/dx for each of the following

1) $x^{-2}y^5 - 2xy^2 + 7x = 12$ **2)** $x + y^2 = e^{x/y}$

3) $\ln y = x + e^y$ **4)** $y^2 = \sin^3 2x + \cos^3 2y$

5) $x^{1+y} + y^{1+x} = 1$

Homework

j. If $x = \tan t - t$, $y = \tan^3 t$, Find y''.

k. If $x = t + \dfrac{1}{t}$, $y = t^2 + \dfrac{1}{t^2}$, Show that $y'' = 2$.

l. If $x = \dfrac{t-1}{t+1}$, $y = \dfrac{t+1}{t-1}$, Show that $y'' = 2y^3$.

m. **Find dy/dx for each of the following**

1) $y^4 - 4x^3y^2 + 6x^2 = 7$ **2)** $\tan^{-1} y = x^2 + y^2$

3) $y = e^{(x+y)^3}$ **4)** $\cosh^{-1}\sec y = xy^3$

<u>Sheet 8 : L'Hôpital's Rule</u>

Lecture Examples

1) $\lim\limits_{x\to\pi/2}\dfrac{2\cos x}{2x-\pi}$ **2)** $\lim\limits_{x\to o}\dfrac{1-\cosh x}{x^2}$

3) $\lim\limits_{x\to o}\dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$ **4)** $\lim\limits_{x\to 1}(1-x)\tan\dfrac{\pi x}{2}$

5) $\lim\limits_{x\to o}\dfrac{x\cos x+\tan 2x}{x\sec x+\sin 4x}$ **6)** $\lim\limits_{\phi\to o}(\operatorname{cosec}\phi-\cot\phi)$

7) $\lim\limits_{x\to\infty}\left(\dfrac{x+3}{x-1}\right)^x$ **8)** $\lim\limits_{x\to o}(\cos x)^{1/x^2}$

Classroom Exercises

1) $\lim_{x\to\pi} \dfrac{1-\sin(x/2)}{\pi - x}$

2) $\lim_{x\to 1} \dfrac{\cot(\pi x/2)}{1-\sqrt{x}}$

3) $\lim_{x\to o} \dfrac{\tan x + \sec x - 1}{\tan x - \sec x + 1}$

4) $\lim_{x\to 0} \dfrac{1-\sqrt{\cos x}}{x^2}$

5) $\lim_{x\to\pi/2} (\sec x - \tan x)$

6) $\lim_{x\to o} \dfrac{\sin 4x - x\cos x}{x\sec x - \tan 3x}$

7) $\lim_{x\to o} (\cos x)^{1/x}$

8) $\lim_{x\to\infty} \left(\dfrac{x}{x+1}\right)^{x}$

Homework

1) $\lim_{x\to 1} \dfrac{1-x^2}{\sin \pi x}$

2) $\lim_{x\to 1} \dfrac{\sin(x^3-1)}{x-1}$

3) $\lim_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$

4) $\lim_{x\to 0} \dfrac{\sin 3x}{1-\cos 4x}$

5) $\lim_{x\to o} \dfrac{\sinh x}{x}$

6) $\lim_{x\to 0} \dfrac{\tanh x}{x}$

7) $\lim_{x\to 0} (1+\sin x)^{1/x}$

8) $\lim_{x\to\infty} \left(\dfrac{x}{x+2}\right)^{x-2}$

Sheet 9 : Partial Differentiation

Lecture Examples

a. Find the first partial derivatives for each of the following

1) $z = (x^2 - y)\sin x^3$

2) $z = (\sin 2y)^x$

b. If $z = \tan^{-1}\dfrac{y}{x}$ **show that**

1) $x\dfrac{\partial z}{\partial y} - y\dfrac{\partial z}{\partial x} = 1$

2) $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x^2} = 0$

c. If $z = f(x^2 + y^2)$ **show that** $x\dfrac{\partial z}{\partial y} - y\dfrac{\partial z}{\partial x} = 0$

Classroom Exercises

d. Find the first partial derivatives for each of the following

1) $z = y^2(x^4-1)^5 + 6y^2x$ **2)** $z = x^2 \sin\sqrt{x} + y\cos(xy)$

3) $z = \tan^{-1}\dfrac{y}{x}$ **4)** $z = e^{x/y}\tanh^{-1}(x^2+y^2)$

e. If $z = \cot^{-1}\dfrac{y}{x}$ show that $\dfrac{\partial^2 z}{\partial y^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

f. If $z = \tan^{-1}\dfrac{x-1}{y-1}$ show that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

Homework

g. Find the first partial derivatives for each of the following

1) $z = x^3 - 3x^2y^4 + y^2$ **2)** $z = (x+y)\sin(x-y)$

3) $z = e^{\frac{y}{x}} \ln\dfrac{x}{y}$ **4)** $z = (1+\sin y)^{1+\cos x}$

h. If $z = \ln(x^2+y^2)$ show that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

i. If $z = \cot^{-1}\dfrac{x}{y}$, show that

1) $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = -1$ and 2) $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

Sheet 10 : Maclaurin's Expansion

Maclaurin's Expansion:

$$f(x) = f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!}\cdots + f^{(n)}(0)\frac{(x)^n}{n!} + \cdots$$

Lecture Examples

a. Find **Maclaurin's Expansion** of each of the following :

(1) $f(x) = \sin 2x$

(2) $f(x) = \ln(2+3x)$, *Find approximate value to* $\ln(2.3)$.

b. Using Maclaurin's Expansion, show that

(1) $e^{-x}\cos x = 1 - x + \frac{1}{3}x^3 - \frac{1}{6}x^4 + \cdots$

(2) $\frac{\cos x}{\sqrt{1+x}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \cdots$

Classroom Exercises

c. Find Maclaurin's Expansion for each of the following

1) $f(x) = \cos 3x$

2) $f(x) = \frac{1}{\sqrt{1+x}}$

3) $f(x) = e^{-3x}$

d. Using Maclaurin's Expansion show that :

1) $e^x \sin x = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} + \cdots$

2) $\frac{e^x}{1-x} = 1 + 2x + \frac{5}{2}x^2 + \frac{8}{3}x^3 + \cdots$

Homework

e. Find Maclaurin's Expansion for each of the following

1) $f(x) = \frac{1}{x+1}$

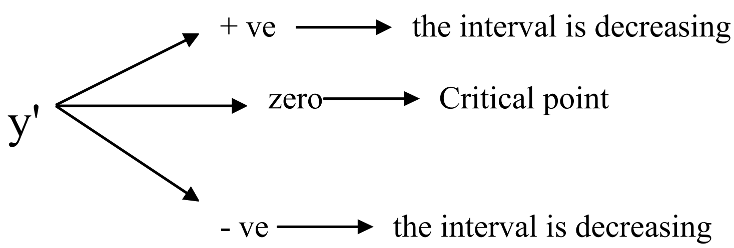
2) $f(x) = \cos 3x$

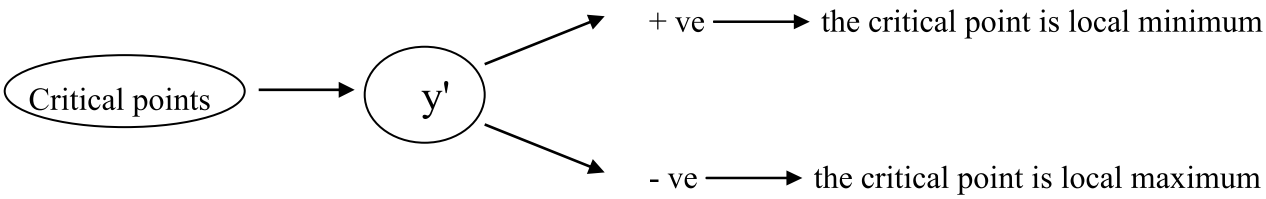
f. Using Maclaurin's Expansion, show that

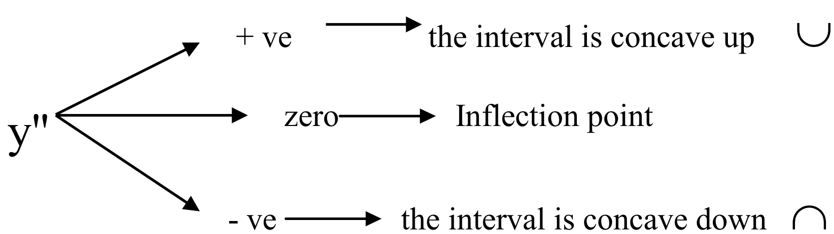
(1) $\frac{e^{-x}}{1-x} = 1 + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$

(2) $\sinh x + \cosh x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$

Sheet 11 : Differentiation applications







Graphing Rational Functions

1. Find the domain of the rational function.
2. Find the vertical asymptote(s) of the rational function.
3. Find the horizontal asymptote of the rational function.
4. Determine the symmetry of the rational function.
5. Find the intercepts of the rational function.
6. Graph the rational function.

1. The **domain** is the set of all **input values** to which the rule applies. These are called your **independent variables**. These are the values that correspond to the first components of the ordered pairs it is associated with.

2. **Vertical Asymptote**

Let $f(x) = \dfrac{P(x)}{Q(x)}$ be written in lowest terms and P and Q are polynomial functions.

If $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$, then the vertical line x = a is a vertical asymptote.

The line x = a is a vertical asymptote of the graph of f(x) if and only if the denominator $Q(a) = 0$ and the numerator $P(a) \neq 0$.
You can have zero or many vertical asymptotes. It will be x = whatever number(s) cause the denominator to be zero after you have simplified the function.

3. **Horizontal Asymptote**

Let $f(x) = \frac{P(x)}{Q(x)}$ be written in lowest terms and P and Q are polynomial functions and $Q(x) \neq 0$.

If $f(x) \to a$ as $x \to \infty$ or $x \to -\infty$, then the horizontal line y = a is a horizontal asymptote.

If there is a horizontal asymptote, it will fit into one of the two following cases:

Case I
If the degree of P(x) < the degree of Q(x), then there is a horizontal asymptote at $y = 0$ (x-axis).

Case II
If the degree of P(x) = the degree of Q(x), then there is a horizontal asymptote at

$$y = \frac{\text{leading coefficient of } P(x)}{\text{leading coefficient of } Q(x)}$$

In other words, it would be the ratio between the leading coefficient of the numerator and the leading coefficient of the denominator.

4. **Determine the symmetry**

The graph is **symmetric about the y-axis** if the function is **even**.

The graph is **symmetric about the origin** if the function is **odd**.

5. **Find any intercepts that exist.**

The ***x*-intercept** is where the graph crosses the x-axis. You can find this by **setting $y = 0$** and solving for x.

The ***y*-intercept** is where the graph crosses the y-axis. You can find this by **setting $x = 0$** and solving for y.

6. **Draw curves through the points, approaching the asymptotes.**

Note that your graph can cross over a horizontal, but it can NEVER cross over a vertical asymptote.

Solved example

$$1)\quad y = f(x) = \frac{x-1}{x^2}$$

Domain :

Our restriction here is that the denominator of a fraction can never be equal to 0. So to find our domain, we want to set the denominator equal to 0 and restrict those values.

let $x^2 = 0$, then x = 0 , hence the domain will be $(-\infty, 0) \cup (0, \infty)$ i.e. Our domain is all real numbers except zero

Intercepts :

y-intercept $\Rightarrow$ to find the y-intercept, we let x = 0 and solve for y $\Rightarrow$ no y-intercept .

x-intercept $\Rightarrow$ to find the x-intercept, we let y = 0 and solve for x $\Rightarrow$ $0 = \frac{x-1}{x^2}$ $\Rightarrow$ x-1 = 0 $\Rightarrow$ x =1 (x-intercept) **.**

Symmetry :

note : f(-x) = f(x) $\Rightarrow$ symmetry about the y-axis $\Rightarrow$ i.e. even function
f(-x) = -f(x) $\Rightarrow$ symmetry about the origin $\Rightarrow$ i.e. odd function

$f(x) = \frac{x-1}{x^2}$, $f(-x) = \frac{-x-1}{x^2}$, So f(-x) $\neq$ f(x) and f(-x) $\neq$ -f(x) $\Rightarrow$ no symmetry .

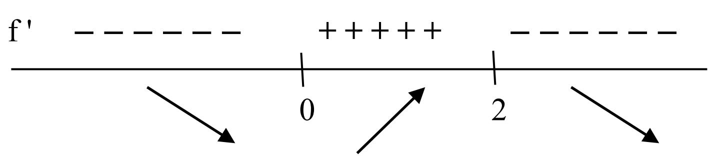
Asymptotes :

1. Vertical Asymptote : $\frac{\text{Not } 0}{0}$, $y = \frac{x-1}{x^2}$, then $\frac{-1}{0}$ $\Rightarrow$ x = 0 is a vertical Asymptote .

2. Horizontal Asymptote: $\lim_{x \to \pm\infty} \frac{x-1}{x^2} = 0$ $\Rightarrow$ y = 0 is a horizontal Asymptote.

Increasing and decreasing intervals :

$$f(x) = \frac{x-1}{x^2} \quad , \quad f'(x) = \frac{2-x}{x^3}$$



Increasing intervals : (0 , 2) , decreasing intervals : ($-\infty$, 0) , (2 , ∞)

Local maximum: at x = 2 **,** **y** = $\frac{1}{4}$ $\Rightarrow$ $(2, \frac{1}{4})$ is a local maximum

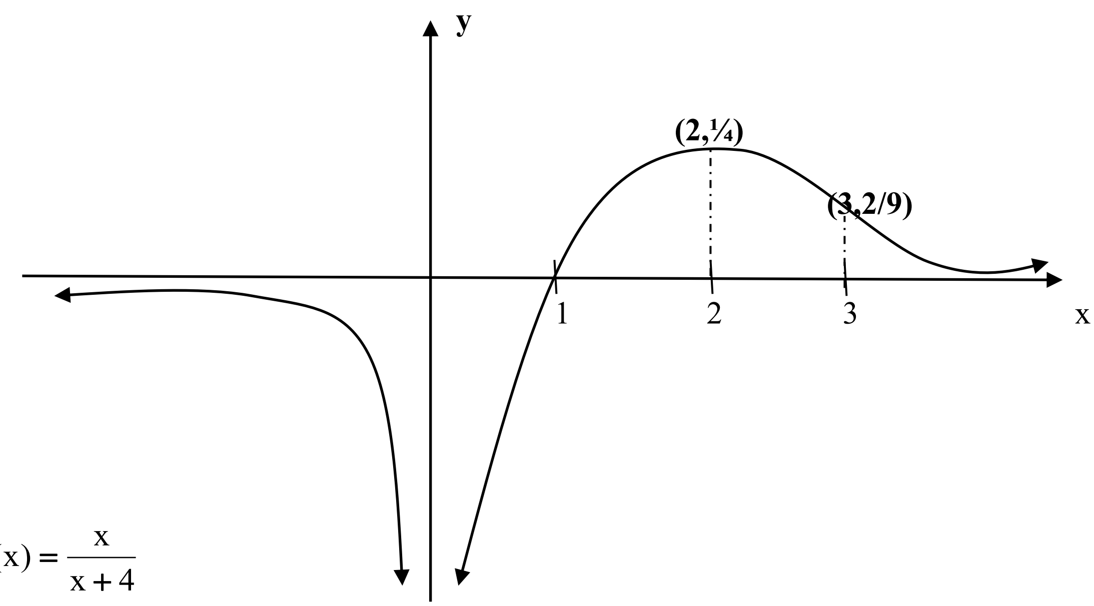
Local minimum: since x = 0 is outside the domain **,** hence no local minimum.

Inflection points : $f'(x) = \frac{2-x}{x^3}$, $f''(x) = \frac{2(x-3)}{x^4}$

f''	− − − − − − − −		− − − − − − − − − − −		+ + + + + + + +
	concave up	0	concave down	3	concave up

again the function is undefined at x = 0 , hence the inflection point is $(3, \frac{2}{9})$

Graph:



2) $y = f(x) = \frac{x}{x+4}$

Domain: let $x + 4 = 0$, then $x = -4$, hence the domain will be $(-\infty, -4) \cup (-4, \infty)$

y-intercept : $y = \frac{0}{0+4} = 0$

x-intercept : $0 = \frac{x}{x+4} \Rightarrow x = 0$

Therefore the function crosses the x-axis and y-axis at the origin (0,0)

Symmetry :

$f(-x) = \frac{-x}{-x+4}$. So $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$ $\Rightarrow$ no symmetry .

Asymptotes :

1. Vertical Asymptote : $f(-4)=\frac{-4}{0} \Rightarrow x=-4$ is a vertical Asymptote.
2. Horizontal Asymptote: $\lim\limits_{x\to\pm\infty}\frac{x}{x+4}=\frac{\text{leading coeff of } x}{\text{leading coeff of } (x+4)}=\frac{1}{1}=1$,

   or

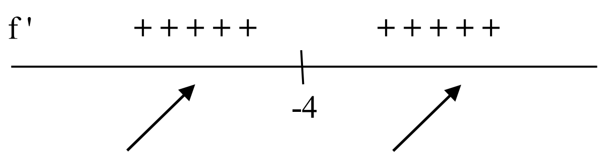
$$\lim_{x\to\pm\infty}\frac{x}{x+4}=\frac{\frac{x}{x}}{\frac{x}{x}+\frac{4}{x}}=\lim_{x\to\pm\infty}\frac{1}{1+\frac{4}{x}}=1 \quad .$$ Hence, y = 1 is a horizontal Asymptote.

Increasing and decreasing intervals :

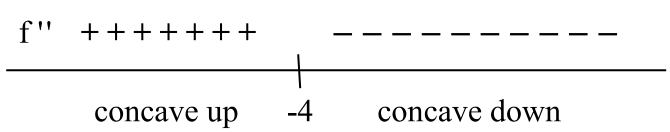
$f(x)=\frac{x}{x+4}$, $f'(x)=\frac{4}{(x+4)^2}$, for $f'(x)=0 \Rightarrow f'(x)$ is undefined

i.e. no solution and no critical points. Hence no local max. and local min exist.

Then let $(x+4)^2=0 \Rightarrow x=-4$.



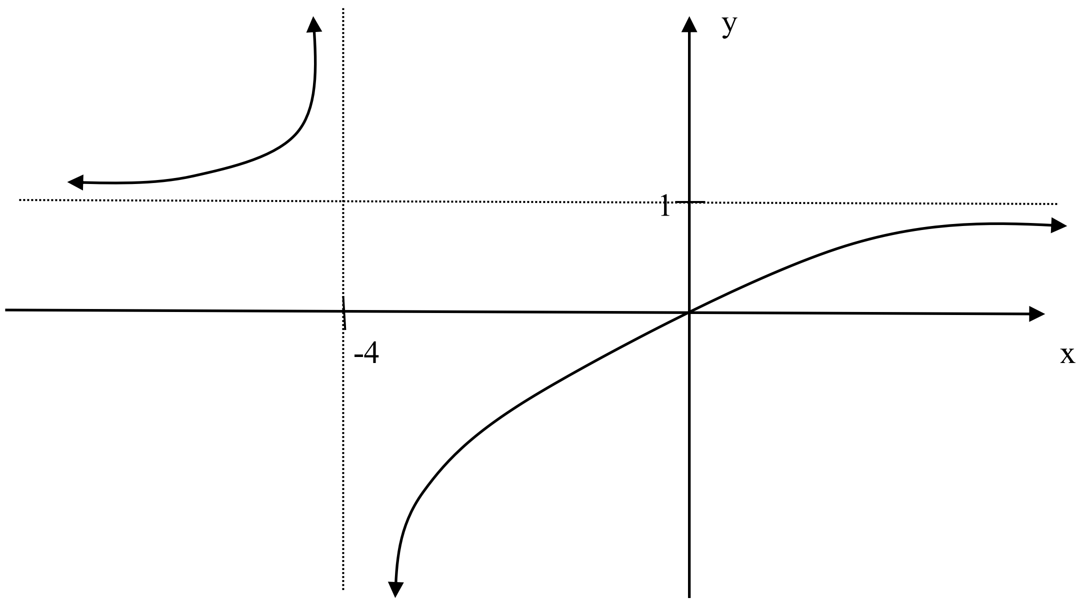
Inflection points : $f'(x)=\frac{4}{(x+4)^2}$, $f''(x)=\frac{-8}{(x+4)^3}$. Again for $f''(x)=0 \Rightarrow f''(x)$ is undefined . Then let $(x+4)^3=0 \Rightarrow x=-4$.



In order to check whether the curve crosses the horizontal Asymptote y = 1 ,

let $\frac{x}{x+4}=1 \Rightarrow x=x+4 \Rightarrow$ no solution $\Rightarrow$ never crosses line y = 1 .

Graph:



The equation of motion of any particle is given by:

- The displacement of the particle:

$$s = s(t)$$

- then, its velocity is given by:

$$v = \frac{ds}{dt}$$

- and its acceleration is given by:

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

Lecture Examples

i) In each of the following curves,
1) $y = x^2 - 4x + 3$ 2) $y = -2x^2 + 12x + 7$

Find,

a. The critical point.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. Sketch the curve.

ii) In each of the following curves,

1) $y = x^3 - 6x^2 + 10$ 2) $y = (x^2 - 9)^2$

Find,

a. The critical points.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. The inflection point.
e. The concavity of the curve.
f. Sketch the curve

iii) In each of the following curves,

1) $y = \frac{5x}{x^2 + 1}$ 2) $y = \frac{x}{x - 2}$

1. Find the domain of the rational function.
2. Find the vertical asymptote(s) of the rational function.
3. Find the horizontal asymptote of the rational function.
4. Determine the symmetry of the rational function.
5. Find the intercepts of the rational function.
6. The critical points.
7. The intervals in which the curve is increasing and decreasing.
8. The local maximum and minimum points.
9. The inflection point.
10. The concavity of the curve.
11. Graph the rational function.

iv) Find the velocity and the acceleration for each of the following

1) $s(t) = t^3 - 6t^2 + 7$ 2) $s(t) = t^3 e^{t^4 - 1}$

Classroom Exercises

i) In each of the following curves,

1) $y = 2x^2 - 8x + 10$ 2) $y = -3x^2 - 12x$

Find,

a. The critical point.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. Sketch the curve.

ii) In each of the following curves,

1) $y = 6x^2 - x^3$ 2) $y = x^3 - 3x^2 - 9x$

Find,

a. The critical points.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. The inflection point.
e. The concavity of the curve.
f. Sketch the curve.

iii) In each of the following curves,

1) $y = \frac{7x}{x^2 + 3}$ 2) $y = \frac{x+1}{x-3}$

1. Find the domain of the rational function.
2. Find the vertical asymptote(s) of the rational function.
3. Find the horizontal asymptote of the rational function.
4. Determine the symmetry of the rational function.
5. Find the intercepts of the rational function.
6. The critical points.
7. The intervals in which the curve is increasing and decreasing.
8. The local maximum and minimum points.
9. The inflection point.
10. The concavity of the curve.
11. Graph the rational function.

iv) Find the velocity and the acceleration for each of the following

1) $s(t) = \sin 5t - \cos 5t$ 2) $s(t) = t^6(1 - \ln t)^4$

Homework

i) In each of the following curves,

1) $y = 12x - 3x^2$ 2) $y = 3x^2 - 6x$

find

a. The critical point.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. Sketch the curve.

ii) In each of the following curves,

1) $y = 2x^3 - 12x^2 + 18x$ 2) $y = x^3 - 9x^2 + 8$

find

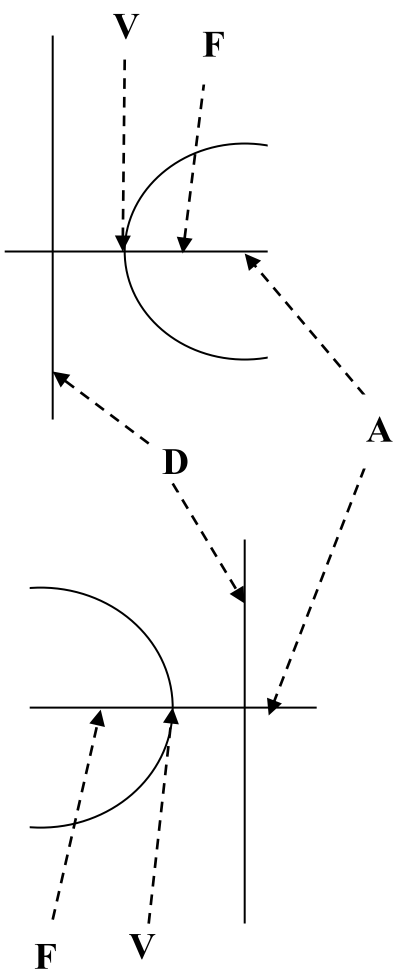
a. The critical points.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. The inflection point.
e. The concavity of the curve.
f. Sketch the curve.

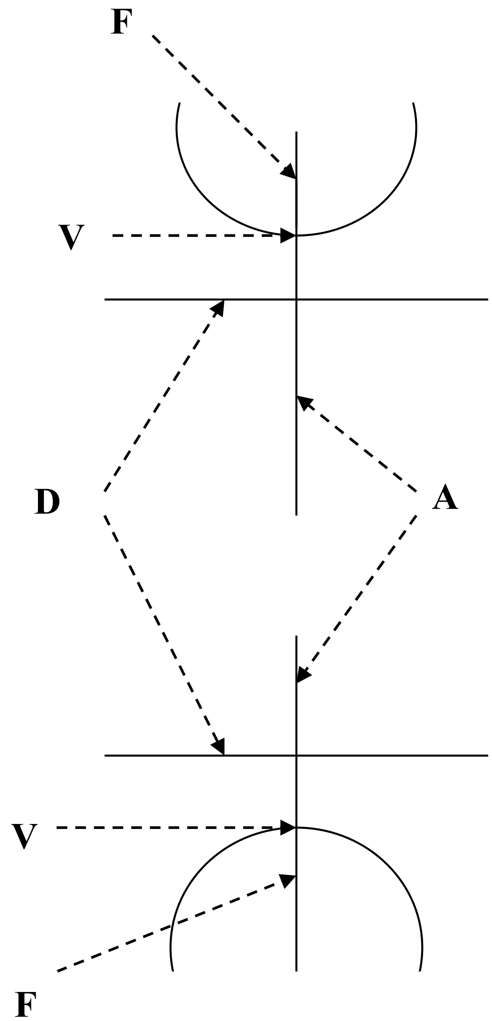
iii) Find the velocity and the acceleration for each of the following

1) $s(t) = t\sinh 3t + \cosh 3t$ 2) $s(t) = \dfrac{1 - e^{2t}}{e^{2t} - e^{2t}}$

Sheet 13 : Conic Sections

The Parabola





$(y - y_0)^2 = 4c(x - x_0)$

vertex	(x_0, y_0)
focus	$(x_0 + c, y_0)$
axis	$y = y_0$
directrix	$x = x_0 - c$

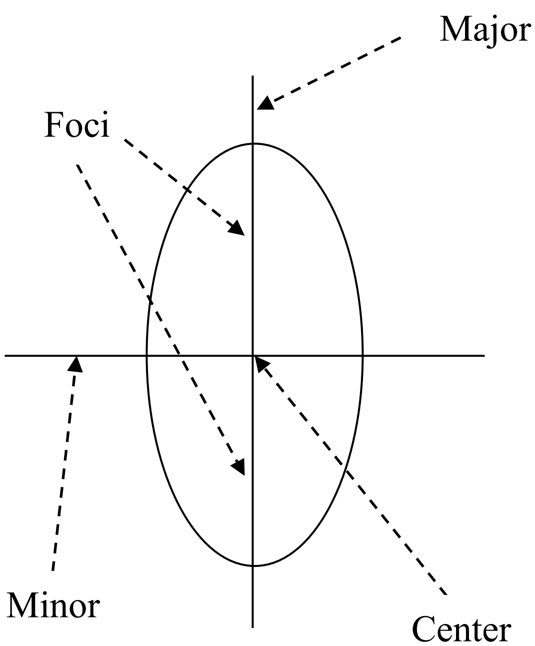
$(x - x_0)^2 = 4c(y - y_0)$

vertex	(x_0, y_0)
focus	$(x_0, y_0 + c)$
axis	$x = x_0$
directrix	$y = y_0 - c$

The Ellipse

General form $\frac{(x-x_0)^2}{b^2}+\frac{(y-y_0)^2}{a^2}=1$

$a > b$



Center (x_0, y_0) , $c^2 = a^2 - b^2$,

major axis = 2a, minor axis = 2b ,

Vertices ($x_0, y_0 \pm a$)

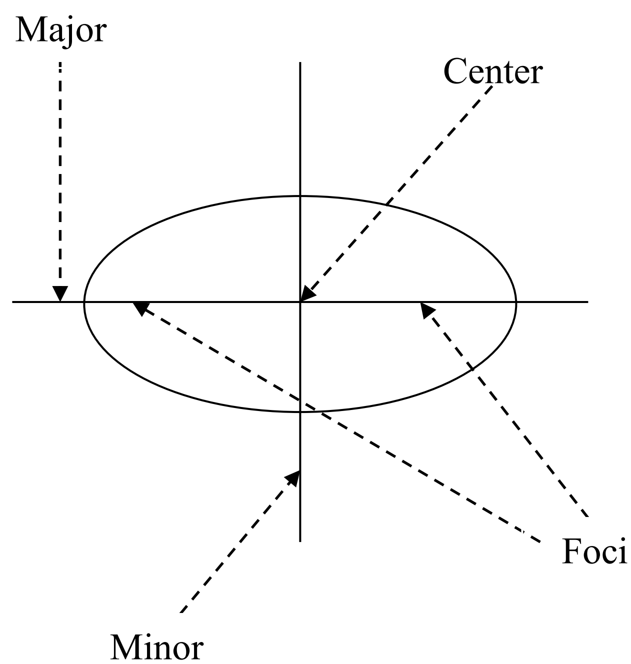
Convertices ($\pm b + x_0$, y_0)

Foci ($x_0, y_0 \pm c$)

Directrix $y = \pm a^2 / c$

General form $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$

$a > b$



Center (x_0, y_0) , $c^2 = a^2 - b^2$

major axis = 2a , minor axis = 2b ,

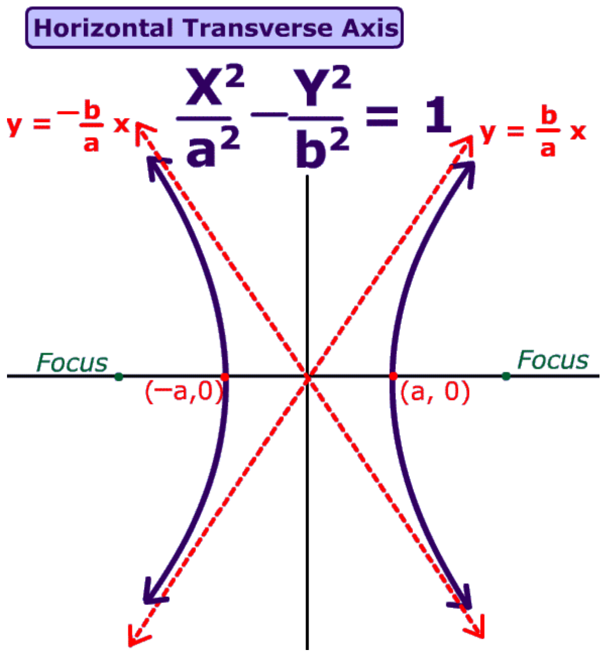
Vertices ($\pm a + x_0$, y_0)

Convertices (x_0, $\pm b + y_0$)

Foci ($\pm c + x_0$, y_0)

Directrix $x = \pm a^2 / c$

The Hyperbola



General form $\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1$

Center (x_0, y_0) , $c^2 = a^2 + b^2$,

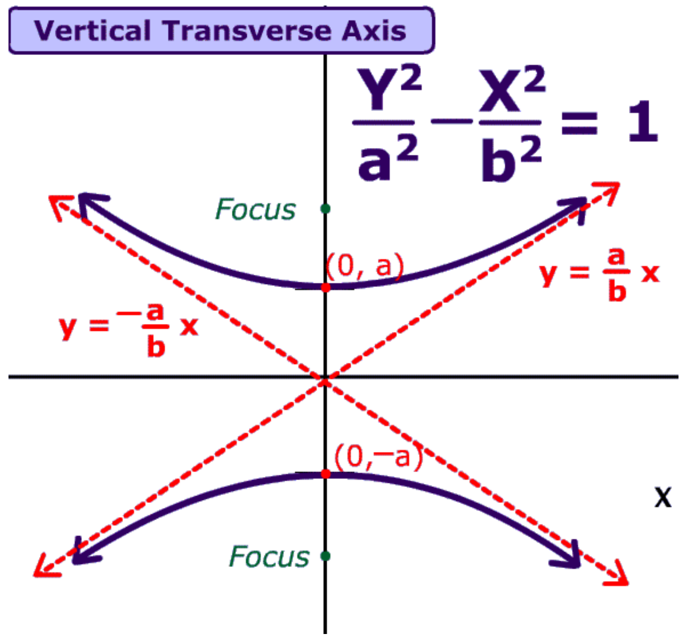
Transverse axis = 2a,
Conjugate axis = 2b ,

Vertices $(\pm a + x_0, y_0)$

Convertices $(x_0, \pm b + y_0)$

Foci $(\pm c + x_0, y_0)$

Directrix $y = \pm a^2/c$



General form $\frac{(y-y_0)^2}{a^2} - \frac{(x-x_0)^2}{b^2} = 1$

Center (x_0, y_0) , $c^2 = a^2 + b^2$

Transverse axis = 2a ,
Conjugate axis = 2b ,

Vertices $(x_0, y_0 \pm a)$

Convertices $(\pm b + x_0, y_0)$

Foci $(x_0, y_0 \pm c)$

Directrix $x = \pm a^2/c$

Lecture Examples

a. Discuss and sketch the following curves:

1) $x^2 + 2x - 4y - 3 = 0$ **2)** $2y^2 - 4x - 4y - 14 = 0$

3) $4x^2 + 9y^2 + 24x = 0$ **4)** $2x^2 + 9y^2 + 8x - 72y + 134 = 0$

5) $9x^2 - 16y^2 - 36x - 32y = 124$

6) $16x^2 - 64x - 4y^2 - 8y - 4 = 0$

Classroom Exercises

b. Discuss and sketch the following curves:

1) $x^2 + 10x + 4y + 13 = 0$ **2)** $y^2 - 4x - 4y + 12 = 0$

3) $x^2 + 4y^2 - 2x - 3 = 0$ **4)** $25x^2 + 16y^2 + 100x - 32y - 284 = 0$

5) $9x^2 - 4y^2 - 72x + 8y + 176 = 0$

6) $-3x^2 + 12x + 2y^2 - 4y = -8$

Homework

c. Discuss and sketch the following curves:

1) $x^2 - 16y - 6x + 9 = 0$ **2)** $y^2 + 6x + 8x - 15 = 0$

3) $16x^2 + 4y^2 - 64x + 8y + 4 = 0$ **4)** $9x^2 + 16y^2 - 36x + 32y = 92$